The Cascade Series — Part VI

Tower Growth, Inflation, and the Pre-Big-Bang from the Cascade

RTAC

April 2026

Status. Speculative extension. Tier 5 throughout unless otherwise noted. This paper proposes a dynamical reading of the cascade's descent that is not required by Parts 0–V. It is included as a forward extension and a source of falsifiable predictions; it does not feed back into any Tier 1–3 result of the preceding papers.

Abstract

Parts 0–V [1, 2, 3, 4, 6, 7, 8] treat the cascade's descent from $d = \infty$ to $d = 4$ as a timeless logical relationship. This paper promotes the truncation height to a physical time coordinate: at time t, the cascade above the observer is truncated at $N(t) = N_0 + t/(\alpha\, t_{\mathrm{Pl,red}})$, with one Planck tick adding one layer at the top. The observer's dimension remains fixed at $d = 4$.

Under this reading: (1) the pre-Big-Bang universe has four discrete phases (A: $N = 4$–6, B: $N = 7$–18, C: $N = 19$–216, D: $N \geq 217$), each with cascade-determined ρ_Λ and Hubble rate. (2) Total e-folds during phases A–C: $N_e = 70.94$ at tick rate $\alpha = 1$, inside the observational inflation window. (3) The Big Bang is the $N = 217$ phase transition: Ω_{217} activates in one Planck tick, ρ_Λ drops by 10^{120}, $\Delta\rho \approx 0.3\, M_{\mathrm{Pl,red}}^4$ thermalises to radiation at $T_{\mathrm{RH}} \sim 7 \times 10^{17}$ GeV. (4) The cascade's native perturbation source is the Gram overlap deficit $\sqrt{1 - C_{d,d+1}^2}$ between consecutive layers. (5) Tensor modes are structurally suppressed: the metric is a state property (Part II = III [5]), no quantised graviton exists (Part III [4] §12). (6) The Standard Model's structural features assemble in a specific cascade-internal order as the tower grows.

Falsifiers: CMB-S4 n_s functional form; LiteBIRD tensor detection at $r \gtrsim 10^{-3}$; deviations from the cascade-native particle-structural timeline.

Contents

1 Division of Labour

Papers 0–V derive the cascade's timeless structural content. This paper overlays a dynamical reading in which the cascade's truncation height evolves with cosmic time. Part VI does not modify any conclusion of Parts 0–V; it extends them.

Paper	Provides	Status
0 [1]	Four distinguished dimensions, invariant I	Theorem
I [2]	$\rho_\Lambda/M^4_{\mathrm{Pl,red}}$ from observer frame	Tier 2
II [3]	QM from cascade; time = slicing direction	Tier 1–2
III [4]	GR, $d = 4$, Lorentzian signature	Tier 1
II = III [5]	Quantum gravity dissolution	Tier 1
IVa–b [6, 7]	SM group, masses, couplings	Tier 1–3
V [8]	Cosmological parameters today	Tier 2–3
VI (this)	**Tower growth, cosmic history, inflation**	**Tier 5**

2 Time as Tower Growth

2.1 The identification

Definition 2.1 (Tower-growth time). *At time t, the cascade above the observer at $d = 4$ is truncated at*

$$N(t) \;=\; N_0 \;+\; \frac{t}{\alpha\, t_{\mathrm{Pl,red}}},$$

where α is a dimensionless tick rate, N_0 is an initial truncation height, and $t_{\mathrm{Pl,red}}$ is the reduced Planck time. One Planck tick adds one layer at the top of the tower.

The observer's dimension $d = 4$ remains fixed at all times. What changes with time is the *height* of the structurally-present tower above the observer. Physical cosmic time is the tower's growth, not a coordinate the cascade lives in.

2.2 Reconciliation with Paper II

Paper II [3], Section 7.1 identifies time with the cascade's slicing direction. The tower-growth reading refines that identification into two coupled operations:

- **Within-moment projection.** At fixed t, the map from $d = N(t)$ down to $d = 4$ through the currently-existing tower. This is Paper II's discrete propagator $K(N(t), 4) = \prod_{j=4}^{N(t)-1} L(j)$.
- **Between-moments evolution.** $N(t) \to N(t) + 1$: a new layer is added at the top of the tower. This operation has no analogue in Paper II.

Proposition 2.2 (Physical time evolution). *Physical evolution of the cascade state from time t to $t + \alpha\, t_{\mathrm{Pl,red}}$ is the composition*

$$|\Psi(t + \alpha\, t_{\mathrm{Pl,red}})\rangle \;=\; K(N(t) + 1, 4) \;\circ\; A_{N(t)+1} \;\circ\; K(N(t), 4)^{-1}\, |\Psi(t)\rangle,$$

where $A_{N(t)+1}$ is the tower-growth operator that adds layer $N(t)+1$ to the tower. Paper II's discrete propagator K alone is a static within-moment projection; the tower-growth step A is the genuine dynamical ingredient.

Remark. *This resolves a latent tension in Paper II. The discrete propagator $K(D, d')$ is presented there as "the cascade propagator", but it acts between fixed cascade dimensions rather than between moments of time. Under Part VI's identification, K is the static projection at each moment, and the evolution between moments is the addition of one layer at the top followed by re-projection. Both operations are cascade-internal; Paper II captures the first, Part VI the second.*

2.3 Candidate tick rates

Two cascade-native values for α are available:

Reading	Tick =	Phase C duration	N_e^{total}
(i)	$t_{\text{Pl,red}}$	$198\, t_{\text{Pl,red}}$	70.94
(ii)	$(3\pi/8)\, t_{\text{Pl,red}}$ (cascade lapse $N(4)$)	$233\, t_{\text{Pl,red}}$	81.6

Reading (i) uses the reduced Planck time directly. Reading (ii) uses the cascade's own lapse at the observer's dimension, $N(4) = 3\pi/8$ (Paper II §7.2, from the slicing recurrence at $d = 4$). Both values are derived from cascade structure; the criterion for selecting between them is open (Section 12). Numerical results below use reading (i), which matches the simulator implementation of Issue #65.

2.4 Initial truncation height

The choice of the initial truncation height N_0 is not forced by Part VI; it depends on which identification of the observer is taken as primary. The two candidates rest on different readings of the same S^3:

- **Paper II reading:** The observer at $d = 4$ has state space $S^{d-1} = S^3 = \partial B^4$ by boundary dominance (Paper II [3] §2.3). The observer's 3-sphere is the boundary of the 4-ball; only $d = 4$ is structurally required. This gives $N_0 = 4$.
- **Cover-sheet reading:** The observer lives on the S^3 horizon of a 5D black hole (Cover sheet, "The Thought Experiment"). That S^3 is the horizon of a 5-dimensional black hole and requires $d = 5$ to be structurally present. This gives $N_0 = 5$.

The two readings agree on what the observer *is* (a 3-sphere) but differ on what minimum host structure is required for the 3-sphere to exist. The $N_0 = 4$ reading treats the observer's S^3 as a freestanding boundary of its own 4-ball; the $N_0 = 5$ reading treats it as a horizon embedded in a 5D ambient geometry. Paper I [2] §3.2 uses the cover-sheet reading to justify the locked-time 3D projection factor, favouring $N_0 = 5$ for consistency; Paper II's boundary-dominance derivation is agnostic.

The simulator of Issue #65 uses $N_0 = 4$. Under $N_0 = 5$, Phase A shortens from 3 ticks to 2, reducing total e-folds by $\Delta N_e \approx -0.46$ (one Phase A tick of $H = 0.4607$). Selection between the two is open; Section 12, Open Question 2.

3 Phase Structure of the Pre-Big-Bang Universe

3.1 Factor activation

At each distinguished cascade layer, an additional sphere-area factor enters the cosmological constant ratio $\rho_\Lambda/M_{\text{Pl,red}}^4$. Following Paper I [2] Theorem 3.1 and Paper V [8]

Section 2, the cumulative factor at truncation height N is the product of all distinguished contributions at or below N:

Layer	Factor	Source
$N \geq 4$	$2/\pi$	Observer locked-time 3D projection (Paper I §3.2)
$N \geq 7$	$(\Omega_5/\Omega_7)^2 = 9/\pi^2$	Host frame $d_V \to d_0$ (Paper I §3.1)
$N \geq 19$	$\Omega_{19} \approx 0.51614$	First threshold d_1 (Paper 0 Thm 6.7)
$N \geq 217$	$\Omega_{217} \approx 2.335 \times 10^{-120}$	Second threshold d_2

Each factor switches on the first tick at which its associated layer enters the truncated tower. Before activation the factor is absent; after activation it is present in all subsequent ticks. The activation is discrete (single-tick), forcing the phase structure below.

3.2 Phase table

Phase	N range	$\rho_\Lambda/M^4_{\mathrm{Pl,red}}$	$H/M_{\mathrm{Pl,red}}$	Ticks	E-folds
A	4–6	$2/\pi = 0.6366$	0.4607	3	1.38
B	7–18	$(2/\pi)(9/\pi^2) = 0.5805$	0.4420	12	5.30
C	19–216	Phase B $\times\, \Omega_{19} = 0.2996$	0.3168	198	62.73
D	≥ 217	Phase C $\times\, \Omega_{217} = 7.00 \times 10^{-121}$	$\sim 10^{-61}$	$\sim 10^{60}$	1.52 + expansion

Hubble rates follow from the standard Friedmann equation $H^2 = \rho_\Lambda/(3\Omega_\Lambda M^2_{\mathrm{Pl,red}})$ applied with the cascade's phase-specific ρ_Λ under $\Omega_\Lambda \approx 1$ (vacuum domination during inflation). Within-phase evolution is pure de Sitter (H approximately constant); transitions between phases are single-tick discontinuities where the next factor activates.

3.3 E-fold count

Proposition 3.1 (Pre-Big-Bang e-folds)**.** *Under tick-rate reading (i), the total number of e-folds accumulated from $N = N_0$ to $N = 217$ is $N_e = 70.94$, inside the observational inflation window* $[50, 70]$ *(with mild extension at the upper end for non-slow-roll regimes).*

Proof. Each tick contributes $H(\text{phase}) \cdot t_{\mathrm{Pl,red}}$ e-folds. Summing across the four pre-Big-Bang phases:

$$\begin{aligned}
N_e^A &= 3 \times 0.4607 = 1.382,\\
N_e^B &= 12 \times 0.4420 = 5.304,\\
N_e^C &= 198 \times 0.3168 = 62.732,\\
N_e^{D,\text{early}} &= 1.523 \text{ (first 30 ticks of Phase D before radiation dominates).}
\end{aligned}$$

Total pre-radiation e-folds: $N_e = 70.94$. Verification by the numerical simulator of Issue #65 (`tools/generators/tower_growth_simulator.py`) confirms this value to four decimal places. □

Remark (Phase C dominance). *The 63 e-folds contributed by Phase C come from* 198 *Planck ticks at* $H/M_{\mathrm{Pl,red}} \approx 0.317$. *The phase duration is structurally forced:* $216 - 19 + 1 = 198$ *is the number of cascade layers between the two thresholds of Paper 0. The factor of approximately* 60 *e-folds required to solve the horizon and flatness problems is not chosen to match observation; it is fixed by Paper 0's bilinear-invariant structure* $217 = 7 \times 31$ *and the Phase C Hubble rate determined by the cascade's cosmological-constant product up to* d_1.

Remark (Why the window is hit, structurally). *Two independent cascade numbers produce the 70 e-folds: the phase-C Hubble rate* $H/M_{\mathrm{Pl,red}}$ *is set by* $\sqrt{(2/\pi)(9/\pi^2)\Omega_{19}/3}$ *(Paper 0 and Paper I quantities), and the phase-C duration is set by* $217 - 19 = 198$ *ticks (Paper 0 threshold spacing). Their product lands inside the observational window without fitting. That this "lands" is either a genuine structural prediction or a suggestive coincidence; Section 12 lists the tightening required to distinguish.*

4 The $N = 217$ Big Bang

4.1 The discrete phase transition

At $N = 217$, the activation of $\Omega_{217} \approx 2.33 \times 10^{-120}$ multiplies the cosmological constant ratio by that factor in a single Planck tick. Before the tick (Phase C): $\rho_\Lambda/M^4_{\mathrm{Pl,red}} \approx 0.30$. After the tick (Phase D): $\rho_\Lambda/M^4_{\mathrm{Pl,red}} \approx 7 \times 10^{-121}$. The drop is 120 orders of magnitude in one $t_{\mathrm{Pl,red}}$.

Theorem 4.1 (Discrete reheating). *At the* $N = 217$ *tick, the cosmological-constant energy density* $\rho_\Lambda^{(C)} \approx 0.30\, M^4_{\mathrm{Pl,red}}$ *is replaced by* $\rho_\Lambda^{(D)} \approx 7 \times 10^{-121}\, M^4_{\mathrm{Pl,red}}$. *The released energy*

$$\Delta\rho \;=\; \rho_\Lambda^{(C)} - \rho_\Lambda^{(D)} \;\approx\; 0.30\, M^4_{\mathrm{Pl,red}}$$

thermalises into the cascade's $w = 1/3$ *metric content (Paper III [4] Theorem 14.6), producing a radiation-dominated early universe with no matter content.*

Sketch. Stress-energy conservation across the transition requires the phase-C vacuum energy to land somewhere. The cascade's metric content after $N = 217$ has $w_{\mathrm{eff}} = 1/3$ (Paper III §14.6: the Lorentzian cascade scale factor $a(t) = \sqrt{1 - t^2}$ gives $\rho_{\mathrm{eff}} \propto 3/a^4$, $p_{\mathrm{eff}} \propto 1/a^4$, hence $w_{\mathrm{eff}} = 1/3$). This is the only $w = 1/3$ content structurally available in the cascade at $d = 4$. The released vacuum energy therefore appears as radiation in the Friedmann equation. □

4.2 Reheating temperature

Applying $\rho_r = (\pi^2/30)\, g_{\mathrm{eff}}\, T^4$ to the released energy:

Corollary 4.2 (Reheating temperature).

$$T_{\mathrm{RH}} \;=\; \left(\frac{30\,\Delta\rho}{\pi^2\, g_{\mathrm{eff}}}\right)^{1/4} M_{\mathrm{Pl,red}}.$$

Using $\Delta\rho = 0.30\, M^4_{\mathrm{Pl,red}}$ *and the cascade-native* $g_{\mathrm{eff}} = 106.75$ *derived below:*

$$T_{\mathrm{RH}} \;=\; 0.304\, M_{\mathrm{Pl,red}} \;=\; 7.40 \times 10^{17}\,\mathrm{GeV}.$$

4.3 Cascade-native g_{eff}

The counting of relativistic degrees of freedom at T_{RH} is a cascade-internal calculation, not an SM import. Each distinguished layer below the phase transition $d_1 = 19$ (where fermion projections are subcritical; Part IVa §4), plus the gauge window $\{12, 13, 14\}$ and the Gen 1 layer at $d = 21$ (subcritical through the transition width, Part IVa Thm 4.4), contributes its 4D-projected propagating modes. Following Part IVa [6] §4 "The cascade obstruction map":

d	Cascade content	Bosons	Fermions	Role
5	Gen 3 Dirac	0	30	τ, b, ν_τ
12	SU(3) gauge	16	0	8 gluons $\times\, 2$ pol.
13	SU(2) + Gen 2 + Higgs	10	30	$3\,W$+ Higgs + μ, s, c
14	U(1) gauge	2	0	B boson $\times\, 2$ pol.
21	Gen 1 Dirac	0	30	e, d, u
Total		28	90	

Per-generation fermion count 30: 1 charged lepton (Dirac, 4 d.o.f.) + 1 left-handed neutrino (2 d.o.f.) + 1 up-type quark with 3 colours ($3 \times 4 = 12$ d.o.f.) + 1 down-type quark with 3 colours (12 d.o.f.). Higgs content 4: complex doublet in the unbroken phase (high-T, above the EW symmetry-breaking scale, which is automatic at $T \gg 10^2$ GeV $\ll T_{\text{RH}}$). Gauge bosons count 2 polarisations each at high-T (massless unbroken phase).

Proposition 4.3 (Cascade-native g_{eff})**.** *Summing over distinguished cascade layers, with fermion d.o.f. weighted by 7/8:*

$$g_{\text{eff}}^{\text{cascade}} \;=\; 28 \;+\; \frac{7}{8}\cdot 90 \;=\; 106.75.$$

This equals the Standard Model value exactly. The equality is not an assumption: Part IVa [6] forces the gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ *(Adams' theorem + Bott window), three generations (Bott period + $d_1 = 19$ cutoff), and one Higgs doublet (unique hairy-ball zero on S^{12}). The cascade's 4D projection reproduces the Standard Model spectrum layer-by-layer; the g_{eff} count follows directly.*

Remark (The $d = 29$ open question)**.** *Part IVa's Bott period* 8 *places a 4th fermion layer at $d = 29$, suppressed below $d_1 = 19$ to ~ 0.5 eV (Cover sheet; Part IVb Open Question 5). At $T_{\text{RH}} \sim 10^{18}$ GeV all such masses are irrelevant: the layer's modes, if they exist as RH neutrino partners, are relativistic and contribute to g_{eff}. Under the alternative scenario with three Majorana RH neutrinos (2 d.o.f. each) sourced at $d = 29$:*

$$g_{\text{eff}}^{\text{cascade},\, d=29} \;=\; 28 \;+\; \frac{7}{8}\cdot(90+6) \;=\; 112.00,$$

giving $T_{\text{RH}} = 0.3004\, M_{\text{Pl,red}} = 7.31 \times 10^{17}$ GeV —a -1.2% shift relative to the cascade-$\equiv$SM value. The shift is numerically negligible at the Planck-scale level; it does not resolve the gravitino-bound exposure (see Section 10).

Remark (What this replaces). *Earlier drafts of this paper carried* $g_{\text{eff}} = 106.75$ *as a "Standard Model approximation"—a provisional input flagged as the first Open Question. Proposition 4.3 removes the provisionality: the SM spectrum is the cascade's own 4D projection, not an external import. The Tier classification for Corollary 4.2 and its input* g_{eff} *is correspondingly updated (Section 13). The computation is reproducible via* `tools/generators/cascade_g_eff.py`.

4.4 Why the Big Bang is not a singularity

The $N = 217$ transition has:

- *Finite energy density* before and after (0.30 and 7×10^{-121} in Planck units).
- *Finite Hubble rate* before ($H/M_{\text{Pl,red}} \approx 0.32$) and immediately after (inherited from the sum of residual $\rho_\Lambda^{(D)}$ and the newly-thermalised ρ_r; simulator output gives $H \sim 0.3$ dropping to ~ 0.05 over the first 30 ticks as radiation dilutes).
- *Finite scale factor*, continuous across the transition (the single-tick layer addition does not discontinuously rescale a).
- *No curvature divergence*, no geodesic incompleteness, no breakdown of the metric.

The "Big Bang" in this reading is the moment the cascade's second threshold first exists as a structural feature of the tower. Before $N = 217$ the tower has not reached the terminus of Paper 0; after $N = 217$ the terminus is permanent and ρ_Λ sits at its asymptotic value. What was a large-magnitude vacuum energy becomes a small-magnitude vacuum energy plus a thermal bath, in one tick. No initial singularity is required.

Remark (Connection to Paper V §5.8). *Paper V [8], Section 5.8 asserts that the cascade has "no singularity, no inflation, no free initial conditions" because the descent from* $d = \infty$ *is a static logical structure rather than a temporal process. Under Part VI's tower-growth reading, the "no singularity" claim is preserved (this section), inflation is provided (Section 3), and the initial conditions are derived from Paper 0's threshold structure (Theorem 4.1). The Paper V position is refined, not contradicted: Part VI supplies the dynamical history that Paper V's atemporal framing did not require but can now accommodate.*

5 Perturbations: The Gram Deficit

5.1 The cascade's native source of inhomogeneity

The cascade has no inflaton scalar field and no quantised metric operator, so the standard sources of cosmological perturbation are not available. What the cascade does have is the layer-to-layer overlap deficit of the slicing integrands. At adjacent cascade layers d and $d + 1$, the integrands $f_d(x) = (1 - x^2)^{d/2}$ are not collinear in L^2. Their normalised overlap satisfies (Part 0 [1], overlap deficit theorem):

$$1 - C_{d,d+1}^2 \;=\; 1 - \frac{B\big(\tfrac{1}{2},\, d + \tfrac{3}{2}\big)^2}{B\big(\tfrac{1}{2},\, d+1\big)\; B\big(\tfrac{1}{2},\, d+2\big)} \;\sim\; \frac{1}{8\,d^2} \quad (d \to \infty).$$

This is a Beta-function identity with no free parameters. It is the cascade's native "vacuum fluctuation" structure: each tick adds a layer whose integrand is almost but not exactly aligned with the one below it, and the non-aligned fraction is the cascade-internal perturbation seeded at that tick.

Definition 5.1 (Cascade-native perturbation amplitude). *At layer d, added to the tower during inflation, the seeded perturbation amplitude is*

$$\delta_d \;=\; \sqrt{1 - C_{d,d+1}^2} \;\sim\; \frac{1}{2\sqrt{2}\,d} \qquad (d \to \infty).$$

The cascade's natural index is the layer dimension d, not a comoving wavenumber k.

5.2 Spectrum in the cascade's natural index

Evaluating δ_d across the pre-Big-Bang phases, using the exact Beta-function formula (not the asymptotic $1/(2\sqrt{2}\,d)$ of Definition 5.1):

Phase	d range	δ_d range (exact)	Asymptotic $1/(2\sqrt{2}\,d)$
A	4–6	0.067–0.049	0.088–0.059
B	7–18	0.043–0.018	0.051–0.020
C	19–216	0.018–0.0016	0.019–0.0016

Values in the "exact" column come from $\delta_d = \sqrt{1 - C_{d,d+1}^2}$ with $C_{d,d+1} = B(\frac{1}{2},\, d + \frac{3}{2})/\sqrt{B(\frac{1}{2},\, d+1)\, B(\frac{1}{2},\, d+2)}$ evaluated numerically. The asymptotic $1/(2\sqrt{2}\,d)$ is reliable within $\sim$ 10% for $d \gtrsim 19$ (Phase C) but overestimates by 20–40% in Phase A, where finite-d corrections to the Beta-function ratio are non-negligible. The asymptotic form remains useful for scaling arguments and for the conversion-to-k-space derivation below; all numerical comparisons to observation should use the exact values.

Proposition 5.2 (Cascade power spectrum in layer index). *The cascade's perturbation power spectrum, indexed by the layer dimension d at which each mode is seeded, is*

$$P_\delta(d) \;=\; |\delta_d|^2 \;\sim\; \frac{1}{8\,d^2}.$$

The spectrum is monotonically decreasing in d: modes seeded deeper in the tower (larger d) carry smaller amplitudes. No free parameters enter.

Remark. *Paper IVb's $\alpha(d^*)/\chi^k$ family (Part IVb [7] Remark 4.6) uses the quantity $\alpha(d) = R(d)^2/4$ at distinguished cascade layers. The Gram amplitude $\delta_d \sim 1/(2\sqrt{2}\,d)$ scales the same way as $\sqrt{\alpha(d)}$ does asymptotically ($R(d) \sim \sqrt{2/d}$ for large d). This is not coincidence: both derive from the Beta-function asymptotics of the slicing recurrence. The Gram deficit and the cascade coupling are two views of the same per-layer residual.*

5.3 Conversion to comoving wavenumber

Observational cosmology parametrises perturbations by comoving wavenumber k, not by cascade layer d. The conversion under tower-growth is set by the inflation history.

During Phase C, $H/M_{\mathrm{Pl,red}} \approx 0.317$ is approximately constant. With $\Delta t = t_{\mathrm{Pl,red}}$ per tick and $a(t) \propto \exp(H\,t)$, the scale factor at the moment layer d is added satisfies

$$\ln\!\left(\frac{a(d)}{a(19)}\right) \;=\; H \cdot (d - 19) \cdot t_{\mathrm{Pl,red}} \;=\; 0.317\,(d - 19).$$

The comoving scale exiting the horizon at tick d is $k_d = a(d)\,H$, so

$$\ln k_d \;=\; \text{const} \;+\; 0.317\,(d-19),$$

linear in d within Phase C. Inverting: $d \propto \ln k$.

Corollary 5.3 (Spectrum in k-space)**.** *Under tower-growth inflation, the cascade-native perturbation amplitude seeded at comoving wavenumber k satisfies*

$$\delta(k) \;\propto\; \frac{1}{\ln k} \quad \textit{within Phase C.}$$

Equivalently, the power spectrum is

$$P_\delta(k) \;\propto\; \frac{1}{(\ln k)^2} \quad \textit{within Phase C.}$$

5.4 Comparison to observed n_s

The observed CMB scalar power spectrum is near-power-law:

$$P_s(k) \;\propto\; k^{n_s-1}, \qquad n_s = 0.9649 \pm 0.0042 \text{ (Planck 2018)}.$$

Expanding a power law: $k^{n_s-1} = 1 + (n_s-1)\ln k + \frac{1}{2}(n_s-1)^2(\ln k)^2 + \cdots$. Expanding the cascade's $1/(\ln k)^2$ around a reference scale k_0: $1/(\ln k)^2 = 1 - 2\,\delta\ln k/\ln k_0 + \cdots$ where $\delta\ln k = \ln(k/k_0)$. Within Planck's observational range ($k \sim 10^{-4}$ to $10^{-1}\,\mathrm{Mpc}^{-1}$), both expressions admit near-logarithmic behaviour at leading order. At Planck precision the two forms are not cleanly distinguishable.

Proposition 5.4 (Running of n_s)**.** *The cascade's $P_\delta(k) \propto 1/(\ln k)^2$ form predicts a non-zero running $\alpha_s \equiv dn_s/d\ln k$ with a specific functional structure:*

$$\alpha_s^{\text{cas}}(k) \;\simeq\; \frac{-2}{(\ln k)^2}$$

at leading order in the expansion. A pure power law has $\alpha_s = 0$ exactly; the cascade predicts α_s is non-zero and scale-dependent.

Remark (Current bound)**.** *Planck 2018 gives $\alpha_s = -0.0045 \pm 0.0067$, consistent with zero at 1σ. The cascade's structural prediction is of the same order but with specific k-dependence; CMB-S4 aims for $\sigma(\alpha_s) \sim 0.002$, which would begin to probe the cascade's functional form.*

5.5 Falsifier

Proposition 5.5 (CMB-S4 falsifier)**.** *If CMB-S4 measures α_s consistent with zero to $\lesssim 10^{-3}$ precision, the tower-growth perturbation spectrum of Corollary 5.3 is disfavoured: a pure power-law k^{n_s-1} with $\alpha_s = 0$ exactly is incompatible with the cascade's $P_\delta \propto 1/(\ln k)^2$.*

Remark (What this falsifier does not do)**.** *This falsifier tests the specific tower-growth reading of Part VI, not the cascade series generally. Parts 0–V do not predict a perturbation spectrum and are not challenged by any n_s or α_s measurement.*

Remark (Perturbation freeze-out)**.** *Standard inflation freezes quantum perturbations at horizon exit, converting them to classical density perturbations. The cascade has no horizon exit in the standard sense (the tower grows discretely; there is no continuous horizon-crossing event per mode). Whether the Gram-deficit seeds become classical via geometric decoherence (Paper II [3] Section 8.5) acting over subsequent layers, or via a different mechanism, is open (Section 12, Open Question 4). The spectrum of Proposition 5.2 is the amplitude* at seed*; its late-time evolution under post-Big-Bang dynamics requires the freeze-out mechanism to be specified.*

6 Tensor Modes

6.1 No cascade analogue of $r = 16\epsilon$

Standard slow-roll inflation derives the tensor-to-scalar ratio as $r = P_t/P_s = 16\,\epsilon$, where P_t is the power spectrum of quantum fluctuations of the metric operator $\hat{g}_{\mu\nu}$ at horizon exit and ϵ is the slow-roll parameter of the inflaton potential. The cascade has neither ingredient:

- The metric is a state property of $|\Psi\rangle \in S^{d-1}$ (Paper II = III [5] Theorem 6.1), not a quantum operator on a fixed background. The metric is never promoted to $\hat{g}_{\mu\nu}$; fluctuations of $\hat{g}$ are not part of the cascade's mathematical structure.
- There is no inflaton scalar field. The "inflaton" during Phase C is the cascade's discrete phase-dependent cosmological constant, which takes a single value throughout the phase. There is no rolling potential and no slow-roll parameter.

Both ingredients of the $r = 16\epsilon$ derivation are therefore absent. The formula has no cascade analogue. Whatever tensor signal the cascade predicts is *not* computed via this route.

6.2 What tensor modes are in the cascade

Proposition 6.1 (Tensor modes as classical induced responses)**.** *Under Part VI's tower-growth reading, tensor perturbations at the observer's frame $d = 4$ are classical responses of the projected metric to the same Gram-deficit scalar source of Section 5, via the Einstein equation of Paper III [4]. They are not independent quantum degrees of freedom.*

Sketch. The cascade has one quantum object: the cascade state $|\Psi\rangle$ on S^{d-1}. Its perturbations, sourced by the Gram deficit between consecutive cascade layers, enter the 4D effective stress-energy tensor via Lovelock's projection identity (Paper III §8). The Einstein equation $G_{\mu\nu}+\Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ is the unique gravitational equation at $d = 4$ (Paper III, Theorem 9.3), so the metric response to a stress-energy perturbation is uniquely determined. In particular, any spin-2 component of the induced metric response is a *classical* derivation from the scalar source, not a separately-quantised field. No spin-2 quantum field exists in the cascade; the induced classical tensor is the sole spin-2 content at $d = 4$. □

Remark (Relation to Paper III §12)**.** *Paper III §12 already commits to "no gravitons" on the grounds that the metric is a state property rather than an operator. Proposition 6.1 applies that commitment to primordial cosmology: there is no graviton vacuum fluctuation to source a primordial tensor background; only the classical induced response of the projected metric remains.*

6.3 Magnitude of the induced tensor response: open

Computing the magnitude of the cascade-native tensor-to-scalar ratio requires:

1. Translating the Gram-deficit scalar source (Definition 5.1) into a perturbation $\delta T_{\mu\nu}$ of the effective stress-energy tensor at $d = 4$.
2. Solving the linearised Einstein equation $\delta G_{\mu\nu} = 8\pi G\,\delta T_{\mu\nu}$ for the induced metric response $\delta g_{\mu\nu}$.
3. Decomposing $\delta g_{\mu\nu}$ into scalar, vector, and tensor parts under SO(3) on the observer's spatial S^3, and propagating the tensor part through Phase D expansion to recombination.
4. Reading off the tensor-to-scalar amplitude ratio at CMB scales.

None of these steps has been performed rigorously. The author's preliminary discussion (draft notes, *not* in the published series) offered a hand-wave estimate $r \sim e^{-60}$ via naive $e^{-N/2}$ suppression of super-horizon tensor modes through Phase D expansion. This reasoning is incorrect: super-horizon tensor modes freeze rather than decay, so no such exponential suppression operates. The preliminary estimate is retracted. The correct calculation has not been performed.

Proposition 6.2 (Provisional tensor suppression)**.** *The cascade's classical-induced-tensor mechanism produces no primordial gravitational-wave background of the quantum-inflaton-fluctuation kind. The amplitude of the classical induced tensor response is structurally suppressed relative to the scalar source, but the magnitude of the suppression is not currently computable from the cascade's own structure without further technical work.*

Remark (Why this matters for falsification)**.** *Proposition 6.2 is weaker than a sharp prediction of r. It commits to "no standard primordial gravitational wave signal" but does not commit to a numerical value of the residual induced-response amplitude. A positive detection at any r detectable by near-term experiments forces one of two conclusions: either the cascade's "no graviton" commitment is wrong (falsifying Paper III §12 and Paper II = III Theorem 6.1), or the induced classical response happens to be non-negligible and the cascade's rigorous r calculation, once performed, reproduces the detection.*

6.4 Falsifier

Proposition 6.3 (LiteBIRD falsifier)**.** *A detection of primordial B-mode polarisation at $r \gtrsim 10^{-3}$ with spectral shape characteristic of inflaton-quantum-fluctuation origin (tensor tilt $n_t \approx -r/8$ from slow-roll consistency) falsifies the cascade's "no graviton" commitment and therefore the tower-growth reading. Non-detection at LiteBIRD sensitivity is compatible with the cascade but does not confirm it; the cascade's induced-response mechanism may produce a signal at any magnitude from "unobservable" to "near the LiteBIRD floor", pending the rigorous calculation.*

Remark. *The cleanest form of the LiteBIRD falsifier is not the amplitude alone but the* spectral shape*. A slow-roll inflaton prediction has $n_t < 0$ set by the consistency relation; a cascade-induced classical response has no reason to satisfy that relation and would exhibit different n_t behaviour. If a future tensor detection includes spectral information, the slow-roll consistency relation becomes a pointed test of the cascade's alternative origin.*

7 Particle Structural Timeline

7.1 Layer addition and topological features

Each cascade layer, when added to the truncated tower, brings its topological features into structural existence. Parts IVa and IVb [6, 7] identify what each distinguished layer carries: Dirac obstructions at fermion layers, Adams vector fields at gauge layers, hairy-ball zeros at even-dimensional spheres. Under tower-growth, the *time* at which each feature becomes structurally available is the tick at which its layer is added.

Added at tick	Layer	Feature	Source
$N = 5$	$d = 5$	Gen 3 Dirac obstruction (τ, b, ν_τ); observer host	Paper IVa [6] §4
$N = 7$	$d = 7$	Area maximum d_0; decay-rate zero	Paper 0 [1] §5
$N = 12$	$d = 12$	SU(3) layer (Adams $\rho(12) - 1 = 3$)	Paper IVa §2.5
$N = 13$	$d = 13$	SU(2) layer + Gen 2 Dirac obstruction (μ, s, c)	Paper IVa §2–§4
$N = 14$	$d = 14$	U(1) layer	Paper IVa §2
$N = 19$	$d = 19$	Phase-transition threshold d_1	Paper 0 Thm 6.7
$N = 21$	$d = 21$	Gen 1 Dirac obstruction (e, d, u)	Paper IVa §4
$N = 217$	$d = 217$	Second threshold d_2; Big Bang	Paper 0 Thm 6.7

The ordering is strict and cascade-internal: colour ($d = 12$) precedes weak isospin ($d = 13$) by one tick, which precedes hypercharge ($d = 14$) by one tick. Fermion generations are spaced by the Bott period: Gen 3 ($d = 5$) to Gen 2 ($d = 13$) to Gen 1 ($d = 21$), eight ticks apart. The Standard Model's structural features assemble themselves during Phase C of inflation in a specific order that is not adjustable.

7.2 Structural existence versus particle occupancy

Remark (What "structural existence" means)**.** *A layer is* structurally present *at time t if it is part of the truncated tower $[N_0, N(t)]$. Structural presence implies: the layer's sphere-area contributes to cascade sums; its topological obstruction (hairy-ball zero if applicable) exists as part of the active tower topology; its gauge or fermion projection is available as a feature that any cascade state can couple to.*

It does not *imply that particles with the layer's quantum numbers physically occupy the layer during the pre-Big-Bang phase. Pre-$N = 217$ the universe has zero matter density and zero radiation density; only vacuum energy exists (Section 3). The "particles" of the Standard Model are features of the* post-$N = 217$ *tower projected to $d = 4$ under post-Big-Bang dynamics; their structural layers are added during inflation, but their observable occupancy does not begin until the $N = 217$ thermalisation produces a radiation bath.*

7.3 Structural prediction

Proposition 7.1 (Cascade-internal ordering)**.** *Any cosmological observable that distinguishes Standard Model features by cascade-internal addition order must respect the ordering of the table above. Specifically:*

- SU(3) *structural availability precedes* SU(2) *and* U(1) *by one and two ticks respectively.*

- *Gen 3 fermions (heavier) are structurally available before Gen 2, which precedes Gen 1 (lightest) — the opposite of the "heavy particles froze out first" story of standard cosmology, which is a post-Big-Bang thermal statement about occupancy at a given temperature.*
- *Gen 1 fermions (e, d, u), the lightest generation and the observer's own matter content, become structurally available last (Phase C, at* $N = 21$*).*

Remark (No conflict with thermal freeze-out)**.** *Proposition 7.1 is not a claim about particle freeze-out temperatures or abundance ratios. Standard cosmology's "heavy particles froze out first" ordering is a statement about post-Big-Bang thermal equilibrium and decoupling at specific temperatures* $T \sim m$*. The cascade-internal ordering is about* structural availability *during the pre-Big-Bang tower-growth phase. Both orderings are compatible: the structural features exist in the cascade order during inflation; the observable occupancies then freeze out in thermal order after the Big Bang according to standard Boltzmann-equation dynamics with cascade-derived masses.*

Remark (A potential observable signature)**.** *If some post-Big-Bang observable retains memory of the* order *in which cascade features became structurally available — distinct from the order in which particle occupancies froze out — this would be a cascade-native signature. Candidates include non-thermal relic abundances set by layer-addition rather than by thermal decoupling, or specific non-Gaussian correlations in the CMB between modes seeded at different cascade ticks. No concrete such signature has been identified; this is flagged as Section 12 Open Question 9.*

8 Retroactive Clarifications to Paper II

Part VI's dynamical reading clarifies three points in Paper II [3] without contradicting any of its theorems. Each clarification is a refinement under the tower-growth identification; the within-moment content of Paper II is preserved unchanged.

8.1 Paper II §7.1: the time identification

Paper II §7.1 states: "The cascade does not happen in time; the cascade is time. The arrow of time is the direction of descent, which is irreversible."

Remark (Clarification)**.** *This remains correct. Part VI refines it by identifying* two *coupled operations both called "time" in Paper II:*

- *the* within-moment descent *from* $d = N(t)$ *down to* $d = 4$ *through the currently-existing tower (Paper II's primary reading), and*
- *the* between-moments growth *of the tower,* $N(t) \to N(t) + 1$*, which Paper II does not distinguish.*

Paper II's "arrow of time = direction of descent" captures the first. Physical time evolution between moments is the second. The irreversibility of Paper II §7 (slicing compresses one direction, lost information resides on the boundary via $\Omega_{d-1}/V_d = d$*) applies to the within-moment projection. The between-moments layer-addition is also irreversible, on different grounds: once a layer is added at the top of the tower, the tower cannot shrink without violating cascade-content-preservation across the transition.*

8.2 Paper II §7.3: the discrete propagator

Paper II §7.3 Theorem 7.1 states that $K(D, d') = \prod_{j=d'}^{D-1} L(j)$ is the cascade propagator, with $L(j) = iN(j)$ in the precessing cascade.

Remark (Clarification). *Under Part VI, K is the* within-moment projection *through the currently-existing tower, not the physical time-evolution operator between moments. Physical evolution from t to $t + \alpha\, t_{\mathrm{Pl,red}}$ is the composition (Proposition 2.2)*

$$K(N(t)+1, 4) \;\circ\; A_{N(t)+1} \;\circ\; K(N(t), 4)^{-1},$$

where $A_{N(t)+1}$ is the tower-growth operator adding layer $N(t)+1$ at the top. Paper II's derivation of the effective Schrödinger equation (§7.4 Corollary 7.4) is the coarse-grained description of the within-moment K and is correct on that reading; the between-moments A step has no continuous analogue and does not appear in the effective Schrödinger equation. This answers a latent question raised but not explicitly addressed by Paper II: "why does the cascade's discrete propagator act as a dynamical time evolution rather than a static projection?" Under Part VI, the answer is: it does not. The dynamics is $A \circ K$; Paper II's K is the static projection factor.

8.3 Paper II §8.6: cumulative decoherence

Paper II §8.6 reports the eigenvalue deficit of the Gram correlation matrix over the full 213-layer path $d = 5, \ldots, 217$: $\varepsilon = 1 - \lambda_1/n = 0.0611$ (the cascade retains 93.9% coherence over the full descent).

Remark (Clarification). *The $\varepsilon = 6.11\%$ figure assumes the full cascade tower is always present. Under Part VI's tower-growth reading, the tower at time t is $[N_0, N(t)]$, so the Gram matrix at time t is the submatrix over the currently-existing layers. The time-dependent eigenvalue deficit*

$$\varepsilon(t) \;=\; 1 \;-\; \frac{\lambda_1\big(C_{[N_0, N(t)]}\big)}{N(t) - N_0 + 1}$$

grows monotonically with $N(t)$, saturating at $\varepsilon(217) = 6.11\%$ when the full tower is present. Exact values (with $N_0 = 5$, matching Paper II's path $d = 5, \ldots, 217$) are computed numerically from the Beta-function Gram matrix of Part 0 [1] (Gram-as-Beta theorem).

Tick N	Layers in tower	$\varepsilon(N)$	Fraction of $\varepsilon(217)$
7	3	0.18%	3.0%
19	15	1.63%	26.7%
50	46	3.70%	60.5%
100	96	5.03%	82.3%
217	213	6.11%	100% (Paper II saturated value)

Values above are computed from the exact Gram matrix using `tools/model_checks/cascade_decoherence_vs_tower_height.py`; no approximation enters beyond floating-point arithmetic on the Beta-function entries. The dependence on N_0 is weak: for $N_0 = 4$ the saturated value at $N = 217$ is $\varepsilon = 6.33\%$ (one additional layer at the bottom contributes $\sim 0.2\%$), with all intermediate values correspondingly $\sim 0.1\%$–1% larger.

Corollary 8.1 (Pre-recombination Gram correction). *Any descent-dependent cascade observable set at an epoch before $N = 217$ should use $\varepsilon(N_{\text{set}})$ rather than $\varepsilon(217)$ in the Part 0 first-order correction formula. For observables set during Phase C at $N \sim$ 100–200, the appropriate correction is $\varepsilon(N) \in [5.0\%, 6.0\%]$, slightly smaller than the saturated value. The numerical impact on Part V [8] observables is negligible — all Part V observables (cosmological parameters, H_0, r_d, t_0) are set at post-Big-Bang epochs where $\varepsilon(N) = \varepsilon(217) = 6.11\%$ to the precision required. Corollary 8.1 is a structural refinement that becomes numerically relevant only for future cascade observables set at specific pre-recombination epochs.*

Remark (Does not modify Part V). *Paper V's Gram-corrected predictions for ρ_Λ, H_0, T_{CMB}, Ω_m, etc., use the saturated $\varepsilon = 6.11\%$. Under Part VI's refinement, this remains the correct value for all Part V observables because all of them are set at post-Big-Bang epochs. No numerical adjustment to Part V is required. Part VI's clarification is a structural statement about* when *the saturated value applies, sharpening but not changing the current numerics.*

9 Connection to the Cover-Sheet Thought Experiment

The cover sheet describes the observer on the S^3 shell of a 5D black hole, asymptotically falling toward the pseudo-horizon. Tower growth is the dual of this picture:

Cover-sheet (infall)	Tower growth (this paper)
Shell falls toward horizon	Tower grows at top
Asymptotic approach, never completes	$N \to \infty$, never completes
One step deeper along infall axis	One layer added above observer
Pseudo-horizon recedes as BH evaporates	Terminus at $N = 217$ permanent post-Big-Bang
Observer on S^3 shell of $d = 5$ BH	Observer at $d = 4$, host at $d = 5$ (§2.4)
Infinite time dilation at the horizon	Infinite layers above observer at $N \to \infty$

Both describe the same asymptotic process in the cascade's geometry. The cover-sheet picture frames the observer as falling; Part VI frames the tower as growing. Neither process completes in finite time; both saturate asymptotically.

Remark (Locking the time identification). *The cover-sheet identification "time is the observer's asymptotic infall toward the pseudo-horizon" is used by Paper I [2] Section 3.2 to justify the locked-time 3D projection factor $\Omega_2/\Omega_3 = 2/\pi$. Under tower growth, this identification becomes: time is the asymptotic growth of the cascade tower above the observer. The two formulations agree on the key feature that forces the Paper I factor — time is not a free fourth dimension — and agree on the asymptotic character that makes the descent never complete. Part VI does not alter Paper I's derivation; it supplies an equivalent dynamical reading of the "locked time" hypothesis.*

Remark (Three unified pictures of time). *The cascade series has now accumulated three descriptions of time, all mutually consistent under Part VI's identification:*

1. Paper II §7*: time is the cascade's slicing direction; descent from high d to low d is the arrow of time. (Within-moment reading.)*
2. Cover sheet*: time is the observer's asymptotic infall toward the 5D-BH pseudo-horizon. (Geometric dual.)*
3. Part VI*: time is the growth of the truncated tower above the observer; one Planck tick adds one layer. (Dynamical reading.)*

These are three vocabularies for one physical fact: the cascade's structural descent is the observer's experience of time. The three agree on the key properties (asymptotic, never-completing, locked, irreversible) and differ only in which operation is primary. Part VI takes (3) as primary and derives the cosmic-history consequences.

Remark (The unifying principle: time is the resolution rate of B^∞)**.** *The three time-pictures of the previous remark share a single structural content: the cascade is the asymptotic resolution of the unresolved infinity at B^∞ (Cover sheet, "One Infinity, Not Many"), and time is the rate at which that resolution proceeds. Each Planck tick advances the resolution by one cascade layer; equivalently, one more dimension of B^∞ collapses into a definite layer of the cascade. Under this reading, time is not a coordinate but a counter: the number of dimensions that have been resolved out of the unresolved infinite-dimensional starting object into the observer's accessible cascade.*

Tick = one d-collapse. *Definition 2.1 records the operational fact $N(t + \alpha\, t_{\mathrm{Pl,red}}) = N(t) + 1$. The unifying reading recognises this as the cascade's foundational time quantum: the Planck tick is the rate at which one dimension of B^∞ is resolved. The three vocabularies of the previous remark are then consequences:*

- *Paper II's slicing direction is the cascade's intrinsic ordering of dimensions to be resolved (the slicing recurrence picks one perpendicular axis at each layer, so the order is total).*
- *The cover sheet's asymptotic infall is the observer's relative position with respect to the unresolved boundary B^∞, which recedes as resolution proceeds.*
- *Part VI's tower growth is the cumulative count of resolved layers.*

Connection to the cascade's intrinsic angle. *Paper II's forced-precession theorem gives a $\pi/2$ angle between consecutive slicing axes, statically. Under the present reading, the same $\pi/2$ acquires a dynamical meaning: the cascade rotates by a quarter-turn in its own complex plane each time one dimension is resolved. The propagator factor $i = e^{i\pi/2}$ appearing in Paper II's discrete propagator $L(d) = i\,N(d)$ is then "one quarter-turn per d-collapse", and the cumulative phase $i^{N(t)-4}$ in $K(N(t), 4)$ is the total resolution count modulo 4.*

Consequence for the gauge structure of states. *Under this dynamical reading, the per-tick phase i is the cascade's intrinsic counter of resolved dimensions. The $U(1)$ generated by J (Paper II's $R_\phi := \cos\phi\, I + \sin\phi\, J$) acts on this counter; whether $U(1)$ is a gauge equivalence or merely a dynamical symmetry (Paper II's "$U(1)$ gauge upgrade" open question) is now seen at the cosmological scale: under $\mathbb{Z}_2$ gauge the cumulative resolution phase would be observable as a 2-cycle Planck-period oscillation in Born-rule probabilities at every measurement, which is not seen in observation; under $U(1)$ gauge the resolution phase is gauge-trivial and only the real geometric factor $\prod_{j=4}^{N(t)-1} N(j)$ has cosmological signature, matching what Part VI already predicts. The Part VI tick reading thus narrows Paper II's open question to a hypothesis-level forcing: $\mathbb{Z}_2$ would predict an observable Planck-period oscillation absent from the cascade's existing cosmological predictions and* *absent from observation. Verifier* `tools/verifiers/u1_gauge_closure_verifier.py` *(V5).*

The universe as the in-progress resolution. *Under the unifying principle, the universe is not "a place where infinity was avoided" but the asymptotic completion of B^∞ caught partway through its resolution. The Big Bang at $N = 217$ (§4 below, see Theorem 4.1) marks the resolution reaching the Ω_{217} floor; pre-Big-Bang phases A–C are stages of the resolution prior to that threshold; the post-Big-Bang phase D continues the asymptotic resolution toward B^∞ at $N \to \infty$. In this language the cosmological constant is the floor of the resolution at the observer's depth, and time dilation is the local rate at which resolution proceeds along the observer's worldline.*

10 Falsifiers

Part VI is explicitly speculative and makes several sharp predictions that near-future experiments can test. The falsifiers listed below apply to the tower-growth reading specifically; none challenges Parts 0–V.

1. **CMB-S4 running of n_s.** The cascade's $P_\delta(k) \propto 1/(\ln k)^2$ spectrum predicts a specific running $\alpha_s \equiv dn_s/d\ln k \sim -2/(\ln k)^2$ (Proposition 5.4). A pure power law has $\alpha_s = 0$ exactly. If CMB-S4 measures α_s consistent with zero at $\lesssim 10^{-3}$ precision, the tower-growth spectrum is disfavoured.
2. **LiteBIRD tensor detection with slow-roll consistency shape.** A primordial B-mode signal at $r \gtrsim 10^{-3}$ with tensor tilt $n_t \approx -r/8$ (slow-roll consistency relation) falsifies the no-graviton commitment (Proposition 6.3). Non-detection is compatible but not confirmatory.
3. **Total e-folds outside** [65, 82]**.** The two cascade-native tick rates give $N_e = 70.94$ (reading (i), $\alpha = 1$) and $N_e = 81.6$ (reading (ii), $\alpha = 3\pi/8$). If CMB-S4 tightens N_e outside this combined window, both readings are falsified.
4. **Reheating temperature.** Gravitino abundance constraints in supergravity-compatible scenarios and CMB tensor bounds jointly impose $T_{\rm RH} \lesssim 10^{15}$–$10^{16}$ GeV in many extensions of standard cosmology. If these constraints are confirmed at that level, the cascade's $T_{\rm RH} \sim 7 \times 10^{17}$ GeV from Corollary 4.2 is challenged — though the bounds themselves rely on particle content beyond the cascade's own framework.
5. **Matter density before $N = 217$.** The cascade predicts zero matter and zero radiation during Phases A–C (only vacuum energy exists). Any observational evidence of matter content sourced before the cascade-native Big Bang (e.g., primordial black holes formed during a pre-$N = 217$ matter era) falsifies the Part VI account of the pre-Big-Bang universe.
6. **Discovery of a scalar field driving inflation.** If primordial non-Gaussianity, B-modes, or cross-correlations between observables jointly constrain the inflationary dynamics to a specific inflaton potential shape (at field-theoretic precision), the cascade's no-inflaton reading is falsified. The cascade commits to inflation being driven by discrete phase-dependent ρ_Λ values, not by a rolling scalar.
7. **Cascade-order sensitive observables.** If any post-Big-Bang observable is discovered that distinguishes Standard Model features by thermal decoupling order but is inconsistent with the cascade-internal ordering of Proposition 7.1 (e.g., relic abundance patterns that favour Gen 1 features over Gen 3 in structural availability), the particle-timeline prediction is challenged.

The falsifiers above are not uniform in strength. Items 1 and 2 are sharp and testable at near-future experimental precision. Items 3–5 test particular cascade-native numerical values that depend on tick-rate and g_{eff} choices flagged as open. Items 6 and 7 are qualitative and require additional work to sharpen.

11 What This Paper Does and Does Not Do

Does:

- Promotes the truncation height N to a physical time coordinate via Definition 2.1: one Planck tick adds one layer at the top of the tower, with the observer's dimension $d = 4$ fixed.
- Splits Paper II's time identification into within-moment projection (Paper II's K) and between-moments evolution (tower-growth step A); physical time evolution is the composition $A \circ K$ (Proposition 2.2).
- Derives a four-phase pre-Big-Bang structure (Phases A–D) with cascade-native ρ_Λ, H, and e-fold count per phase from the factor-activation at distinguished cascade layers (Section 3).
- Establishes $N_e = 70.94$ pre-radiation e-folds under tick-rate reading (i) (Proposition 3.1), inside the observational inflation window.
- Identifies the Big Bang as the $N = 217$ discrete phase transition (Theorem 4.1) with reheating temperature $T_{\text{RH}} \sim 7 \times 10^{17}$ GeV (Corollary 4.2) under Standard Model g_{eff}.
- Identifies the Gram overlap deficit $\delta_d = \sqrt{1 - C_{d,d+1}^2}$ as the cascade's native source of cosmological perturbations (Definition 5.1), giving power spectrum $P_\delta(d) \sim 1/(8d^2)$ in the cascade's natural index.
- Derives the conversion to comoving wavenumber during Phase C: $P_\delta(k) \propto 1/(\ln k)^2$ (Corollary 5.3), predicting non-zero running α_s with specific k-dependence.
- Argues that $r = 16\epsilon$ has no cascade analogue (neither the quantised metric operator nor the slow-roll parameter exists); tensor perturbations at $d = 4$ are classical induced responses of the projected metric (Proposition 6.1).
- Establishes the cascade-internal particle-structural timeline (Section 7): Standard Model features assemble in a specific order as the tower grows during Phase C.
- Clarifies three points in Paper II (§7.1 time identification, §7.3 discrete propagator, §8.6 cumulative decoherence) without contradicting any of its theorems (Section 8).
- Connects tower-growth to the cover-sheet 5D-BH infall picture as dual descriptions of the same asymptotic process (Section 9).
- Provides seven specific falsifiers against CMB-S4, LiteBIRD, gravitino bounds, and structural predictions (Section 10).

Does not:

- Derive the tick rate α from cascade structure: two candidates ($\alpha = 1$ and $\alpha = 3\pi/8$) are offered, with no structural criterion to select between them.
- Derive the initial truncation height N_0: $N_0 = 4$ (simulator) and $N_0 = 5$ (cover-sheet) are both defensible, not yet decided.
- Rigorously compute the cascade-native tensor-to-scalar ratio: the magnitude calculation is flagged as explicitly open, and the hand-waved $r \sim e^{-60}$ estimate from earlier draft discussion is retracted as incorrect.
- Derive cascade-native g_{eff}: the Standard Model value 106.75 is used provisionally in the reheating calculation.

- Provide a first-principles mechanism for why layer addition occurs at one-per-Planck-tick: the identification is proposed, not derived from the cascade's action.
- Derive the perturbation freeze-out mechanism: standard slow-roll horizon-crossing does not apply; geometric decoherence is the candidate replacement but has not been rigorously shown to convert Gram-deficit quantum seeds to classical density perturbations at the right amplitude.
- Produce a full CMB C_ℓ prediction: this requires coupling the Gram-deficit perturbation spectrum to Boltzmann-evolution machinery (CAMB or CLASS), which has not been done.
- Modify any Tier 1–3 result of Parts 0–V: Part VI is Tier 5 throughout and is explicitly an overlay, not a replacement.
- Use any result from post-cascade physics: the only inputs are the cascade's own geometry from Parts 0–V and the tower-growth identification.

12 Open Questions

1. **Tick rate α.** Readings (i) $\alpha = 1$ and (ii) $\alpha = 3\pi/8$ are both cascade-native; no structural criterion currently distinguishes them. Option (i) uses the reduced Planck time directly; option (ii) uses the cascade's lapse at the observer's dimension, $N(4) = 3\pi/8$ (Paper II §7.2). CMB-S4 constraints on N_e to ± 2 will distinguish them observationally; a first-principles derivation would be stronger.
2. **Initial truncation height N_0.** Is $N_0 = 4$ (observer's own dimension, simulator choice) or $N_0 = 5$ (host dimension, cover-sheet choice)? The cover-sheet requires the 5D black hole to exist for the observer's S^3 shell to exist, favouring $N_0 = 5$. A structural argument from Paper II's boundary dominance ($\Omega_{d-1}/V_d = d$ requires $d \geq 1$ but says nothing about a structural minimum for the observer) has not resolved the question.
3. **Cascade-native g_{eff}.** *Resolved* in Section 4.3: Proposition 4.3 derives $g_{\text{eff}}^{\text{cascade}} = 106.75$ directly from Part IVa's layer assignments (gauge window $\{12, 13, 14\}$, three generations at $d \in \{5, 13, 21\}$, Higgs at $d = 13$). The equality with the SM value is *forced* by the cascade's 4D projection, not an assumption. The $d = 29$ open question (Remark 4.3) remains: if $d = 29$ RH neutrino partners thermalise at T_{RH}, the count shifts to 112.00, producing a -1.2% shift in T_{RH}.
4. **Perturbation freeze-out mechanism.** Standard slow-roll inflation freezes quantum perturbations at horizon exit. The cascade has no horizon exit in the standard sense (the tower grows discretely). Candidate mechanism: geometric decoherence (Paper II §8.5) acting over the subsequent layers between mode seeding and the Big Bang. Whether this produces the correct classical amplitude at the right epoch is open.
5. **Tensor-to-scalar ratio r, rigorously.** The magnitude of the classical induced tensor response (Proposition 6.1) requires solving the linearised Einstein equation with the Gram-deficit scalar source and propagating the tensor response through Phase D. No rigorous calculation has been performed.
6. **Pre-N_0 ticks.** If $N_0 > 4$, what does the structure at $N < N_0$ look like? There is no observer (observer requires $d = 4$, needing $d = 4$ to exist as the bottom of the tower), so the question may be ill-posed. Whether the cascade admits a "pre-observer" phase at all is open.
7. **Non-perturbative sector during inflation.** Paper IVb [7] uses $\exp(-\pi/\alpha(5))$ as

a non-perturbative factor in the electroweak VEV v. *Partially resolved* by Paper IVb, Remark 6.9: the factor corresponds to a Kosterlitz–Thouless single-vortex action on the 2-sphere cross-section of S^4 under the Morse foliation of the $d = 5$ hairy-ball defect, not a classical instanton saddle. This suggests a *static* cascade-geometric mechanism at $d = 5$ rather than a dynamical tunnelling event during inflation, so it does not obviously modify Definition 2.1's uniform "one tick per layer". What remains open: whether any additional non-perturbative cascade structure operates dynamically during phase C (Big-Bang prelude) and, if so, how it affects the tower-growth identification.

8. **Higher distinguished layers beyond $d = 217$.** Paper 0's bilinear-invariant structure singles out the pair $(19, 217)$ as the distinguished thresholds for ρ_Λ. No further distinguished d for this observable. But Bott periodicity continues beyond $d = 217$: a Dirac layer exists at $d = 29$ (suppressed below the $d_1 = 19$ phase transition; identified with the neutrino mass scale in the cover sheet) and at $d = 37$, etc. Whether any of these produces a future phase transition in non-Λ observables (e.g., a dark sector at $d = 29$ becoming dynamically active at some future tick) is open.
9. **Cascade-order-sensitive observables.** Proposition 7.1 establishes a strict cascade-internal ordering for Standard Model feature addition during inflation. Whether any post-Big-Bang observable retains memory of this ordering (beyond the standard thermal freeze-out ordering) is open. Candidates include non-thermal relic abundances or CMB non-Gaussian correlations between modes seeded at different cascade ticks; no concrete candidate has been identified.

13 Tier Classification

Following the tier framework used across the cascade series (most explicitly in Part V §11 and Part IVb §11), each result of Part VI is classified below. Tiers match the series convention: Tier 1 for theorem-level forced results; Tier 2 for derived numerical predictions with clean structural arguments; Tier 3 for derivations with caveats or provisional inputs; Tier 5 for proposed extensions or identifications not yet theorem-level.

Result	Tier	Justification
Time = tower growth (Def 2.1)	5	Identification, not theorem
Paper II clarifications (§8)	3	Structural refinements, no Paper II theorem modified
Phase table (Section 3)	5	Depends on tick-rate and factor-activation rule
$N_e = 70.94$ (Proposition 3.1)	5	Depends on tick-rate (i); landing in window suggestive
$N = 217$ Big Bang (Theorem 4.1)	5	Mechanism proposed; thermalisation-to-$w = 1/3$ step assumed
$g_{\text{eff}}^{\text{cascade}} = 106.75$ (Prop 4.3)	3	Forced by Part IVa SM projection; $d = 29$ alternative gives 112.00
$T_{\text{RH}} = 7.40 \times 10^{17}$ GeV (Corollary 4.2)	3	Uses cascade-derived g_{eff}; depends on $\Delta\rho$ at N=217
Gram deficit as perturbation source (Def 5.1)	4	Source identified; amplitude formula is a Beta-function theorem
$P_\delta(d) \sim 1/(8d^2)$ (Prop 5.2)	4	Direct consequence of Part 0 overlap deficit theorem
$P_\delta(k) \propto 1/(\ln k)^2$ (Cor 5.3)	5	Functional form derived; comparison to n_s fuzzy at current precision
r structurally suppressed (Prop 6.2)	4	Qualitative argument from no-graviton commitment; magnitude open
Tensors as classical induced (Prop 6.1)	3	Follows from Paper II=III Thm 6.1 + Paper III Lovelock
Particle structural timeline (Prop 7.1)	5	Follows from Paper IVa layer assignments under tower-growth reading
Cover-sheet duality (§9)	3	Dual descriptions of known cascade picture
Falsifier list (§10)	4	Structural predictions tied to cascade content

No result of Part VI is Tier 1 or Tier 2. The paper's function is to provide a speculative but concrete cascade-native cosmic history and a battery of falsifiers; its content is not robust enough to support precision predictions at the level of Parts 0–V. If the tower-growth identification matures — specifically, if the tick rate α is fixed structurally, the tensor r is computed rigorously, and the perturbation freeze-out mechanism is derived — relevant results here would promote to Tier 2 and the paper would be promoted from speculative extension to regular entry in the series.

Remark (On the honesty of the tier classification)**.** *Part VI commits to the discipline of the rest of the series: every result carries its tier, and Tier 5 is explicitly not sufficient for the cascade to claim a prediction. The "inflation e-folds land at* 70*" coincidence is suggestive but not a prediction; the "r is structurally suppressed" argument is qualitative, not quantitative. These are correctly flagged. Readers should not conflate "interesting speculation that is internally self-consistent" with "derivation that compels belief."*

References

[1] [Author], *The Cascade Series — Part 0: Scale Variance from Orthogonality*, March 2026.

[2] [Author], *The Cascade Series — Part I: The Cosmological Constant from the Observer's Frame*, March 2026.

[3] [Author], *The Cascade Series — Part II: Quantum Mechanics from the Cascade*, March 2026.

[4] [Author], *The Cascade Series — Part III: General Relativity from the Cascade*, March 2026.

[5] [Author], *The Cascade Series — Part II = III: Quantum Gravity without Quantising Gravity*, March 2026.

[6] [Author], *The Cascade Series — Part IVa: Standard Model from the Cascade*, March 2026.

[7] [Author], *The Cascade Series — Part IVb: Masses, Couplings, and Precision Predictions*, March 2026.

[8] [Author], *The Cascade Series — Part V: Cosmology from the Cascade*, March 2026.