The Cascade Series — Part V

Cosmology from the Cascade: ΛCDM Parameters, the Hubble Constant, and the DESI BAO Observations

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Abstract

The cascade series [1, 2, 3, 4, 5] tests one hypothesis: the infinite-dimensional unit ball, descended to four dimensions, is indistinguishable from our universe. This paper derives the background cosmological parameters of the ΛCDM model from the cascade's geometry, with zero free parameters.

The cosmological constant ratio $\rho_\Lambda/M^4_{\mathrm{Pl,red}} = 18\,\Omega_{19}\Omega_{217}/\pi^3$ is established in Part I through the cascade's bilinear invariant combined with the observer's host frame ($9/\pi^2$ at $d_V \to d_0$) and the cube–sphere bridge ($\pi/2$ at the observer's spatial dimension $d = 3$). This paper inherits that identification and derives the remaining background parameters as functions of π alone. (1) *Dark energy equation of state.* The cascade predicts $w = -1$ exactly. (2) *Matter fraction.* $\Omega_m = 1/\pi = 0.31831$ (observed 0.315, +1.1%), derived from the lapse factorisation $N(d) = \sqrt{\pi}\,R(d)$; the Bott partition gives the subleading-corrected value $\Omega_m^{\mathrm{Bott}} = 0.31150$ (−1.1%), consistent with the descent systematic. Both derivations are reported; they are successive approximations. (3) *Baryon and dark matter fractions.* $\Omega_b = 1/(2\pi^2) = 0.0507$ (+2.8%), $\Omega_{\mathrm{DM}} = (2\pi - 1)/(2\pi^2) = 0.2677$ (+1.4%). (4) *Radiation density.* $\Omega_r = 1/(4\pi^7) = 8.28 \times 10^{-5}$, from the cascade's interior content propagated through seven geometric steps to the gauge boundary. (5) *Hubble constant.* $H_0 = 66.78$ km/s/Mpc from the Friedmann equation with $\rho_\Lambda/M^4_{\mathrm{Pl,red}} = (2/\pi)I$, sitting 0.9% below Planck's 67.4 and 8.5% below the SH0ES local measurement. (6) *CMB temperature.* $T_{\mathrm{CMB}} = 2.642$ K (observed 2.7255 K, −3.07%; descent-dependent population, shares the common sign with the other H_0-scaled quantities; closed to sub-percent by the Gram first-order correction applied to H_0). (7) *Universe age.* $t_0 = 13.88$ Gyr from the flat-ΛCDM integral with cascade $(H_0, \Omega_m, \Omega_\Lambda)$, matching observed 13.80 ± 0.02 Gyr to +0.58%. (8) *Matter–radiation equality.* $z_{\mathrm{eq}} = 4\pi^6 = 3846$. The complete Friedmann equation $H^2(z) = H_0^2[(1 + z)^4/(4\pi^7) + (1 + z)^3/\pi + (\pi - 1)/\pi]$ has every coefficient determined by π.

Under these parameters the cascade fits DESI DR2 BAO with $\chi^2/n = 2.35$ (leading) or 2.23 (Bott), somewhat worse than Planck ΛCDM at $\chi^2/n = 1.90$; the difference is driven by the same two shared outliers (the $z = 0.510$ D_H/r_d bin and the $z = 0.706$ D_M/r_d bin) that also stress ΛCDM. With cascade $r_d \approx 147.7$ Mpc essentially equal to Planck's 147.6 Mpc, the cascade predicts $w = -1$ exactly as a structural theorem with no free parameters; the apparent DESI preference for

$w \neq -1$ is therefore a challenge for both the cascade and ΛCDM to the same degree, and cannot be attributed to a ruler mismatch.

Contents

1 Division of Labour

Papers I–IVb of the cascade series derive, from the cascade's geometry alone: a geometric invariant $I \approx 10^{-120}$ matching the cosmological constant [1]; the structural framework of quantum mechanics [2]; general relativity with $\Lambda = I$, $d = 4$, and Lorentzian signature [3]; and the Standard Model gauge group, symmetry breaking, fermion generations, masses, and coupling constants [4, 5]. This paper derives the background cosmological parameters and the cascade's account of the DESI observations.

Parameter	Cascade source	Section
$\rho_\Lambda/M^4_{\mathrm{Pl,red}} = 18\Omega_{19}\Omega_{217}/\pi^3$	Observer-corrected cascade invariant	From [1]
$w = -1$	Fixed Λ; GB corrections vanish	§3
$\Omega_k \approx 0$	S^3 topology	§4
$\Omega_m = 1/\pi$	Lapse factorisation	§5.1
$\Omega_m^{\mathrm{Bott}} = 0.31150$	Bott partition (subleading)	§5.9
$\Omega_\Lambda = (\pi - 1)/\pi$	Complement of matter fraction	§5.1
$\Omega_b = 1/(2\pi^2)$	Observer's boundary area	§5.2
$\Omega_r = 1/(4\pi^7)$	Interior content, 7 geometric steps	§5.3
$H_0 = 66.78$ km/s/Mpc	Friedmann equation with $\rho_\Lambda = (2/\pi)I\, M^4_{\mathrm{Pl,red}}$	§6
$T_{\mathrm{CMB}} = 2.642$ K	From Ω_r, $M_{\mathrm{Pl,red}}$, H_0, 3 generations	§5.4
$z_{\mathrm{eq}} = 4\pi^6$	Ω_m/Ω_r	§5.5
$r_d \approx 147.7$ Mpc	Derived from cascade parameters	§8
$t_0 = 13.88$ Gyr	Flat-ΛCDM integral	§7

2 The Cosmological Constant

Part I [1] derives the observed cosmological-constant ratio in reduced-Planck units by combining the cascade bilinear invariant with two observer-frame corrections:

$$\frac{\rho_\Lambda}{M^4_{\mathrm{Pl,red}}} = \underbrace{\frac{2}{\pi}}_{\substack{\text{observer bridge}\\ d=3}} \cdot \underbrace{\frac{9}{\pi^2}}_{\substack{\text{host frame}\\ d_V \to d_0}} \cdot \Omega_{19}\,\Omega_{217} = \frac{18\,\Omega_{19}\,\Omega_{217}}{\pi^3} = 6.996 \times 10^{-121}.$$

The cascade bilinear invariant $I = 9\,\Omega_{19}\Omega_{217}/\pi^2 = 1.0990 \times 10^{-120}$ is a Gamma-function theorem (Part 0); the observer-frame factors ($9/\pi^2$ from the host-frame rescaling and $2/\pi$ from the observer's cube–sphere bridge at the 3-dimensional spatial slice) are also Gamma-function identities, not fits. Every numerical prediction of this paper inherits this identification.

3 Dark Energy Equation of State: $w = -1$ Exactly

Theorem 3.1 (Equation of state)**.** *The cascade predicts $w = -1$ exactly.*

Proof. $\Lambda = I$ is a fixed geometric constant, determined entirely by π through the cascade's threshold structure ([1], Theorem 8.10). A fixed cosmological constant has $\rho_\Lambda = \mathrm{const}$, so $p_\Lambda = -\rho_\Lambda$, giving $w = p/\rho = -1$ by definition. No time-evolution mechanism exists in the cascade for Λ: the thresholds $d_1 = 19$ and $d_2 = 217$ are permanent features of the Gamma function, not snapshots of a dynamical field. The lapse function $N(4) = 3\pi/8$ is a fixed Gamma-function value, not a cosmological-time-evolving quantity. □

3.1 Gauss–Bonnet stability of $w = -1$

The cascade geometry at $d = 5$ has S^3 cross-sections that are totally umbilic: $K_{ij} = -(x/\sqrt{1-x^2})\, h_{ij}$ exactly, so $K_{ij} = (K/3)h_{ij}$. For totally umbilic hypersurfaces, the leading Gauss–Bonnet boundary terms cancel identically:

$$2K_{ij}K^{ij}K - \tfrac{2}{3}K^3 = \tfrac{2}{3}K^3 - \tfrac{2}{3}K^3 = 0.$$

The residual GB term $2K\tilde{R} - 2K_{ij}\tilde{R}^{ij} \propto -24x/(1-x^2)^2$ is odd in x; the cascade measure $(1-x^2)^{3/2}$ is even. The integral vanishes exactly:

$$\int_{-1}^{1} (1-x^2)^{3/2} \cdot \frac{-24x}{(1-x^2)^2}\, dx = -24 \int_{-1}^{1} \frac{x}{\sqrt{1-x^2}}\, dx = 0.$$

Corollary 3.2. *$w = -1$ receives zero Gauss–Bonnet correction at leading order, by two independent mechanisms: (i) totally umbilic cancellation from the cascade's spherical symmetry; (ii) parity of the residual integrand under the symmetric slicing. Both follow from the same orthogonality axiom that generates the cascade. The prediction is structurally protected, not merely numerically small.*

4 Spatial Curvature

Theorem 4.1. *The cascade predicts a closed universe (S^3 topology) with curvature radius far exceeding the Hubble radius, giving $\Omega_k \approx 0$ to observational precision.*

Proof. The observer at $d = 4$ lives on S^3, the boundary of B^4. The cascade's slicing recurrence produces round spheres at every step. The curvature radius is set by $a(t)$, which in the Λ-dominated era grows exponentially. For $\Omega_\Lambda \approx 0.68$, the curvature contribution is suppressed to undetectable levels. □

Observed: $\Omega_k = 0.001 \pm 0.002$ (Planck 2018 [6]). Consistent.

5 Cosmological Initial Conditions from the Cascade

The physical identification hypothesis (Definition 2.1 of [3]) holds that the cascade's geometry is our physics. Sphere area is the only independent cascade quantity at each level (Corollary 3.2 of [1]). Geometry is energy: the density fractions derived below are not predictions *about* radiation, matter, and vacuum energy — they *are* the geometric content of the cascade at different distances from the observer. The labels are what the four-dimensional observer, constrained by Lovelock uniqueness to interpret all content through the Friedmann equation, calls them.

5.1 Density fractions from π

The cascade's lapse function factorises as $N(d) = \sqrt{\pi}\, R(d)$, where $R(d) = \Gamma((d+1)/2)/\Gamma((d+2)/2)$. This factorisation separates the cascade's content into two sectors: the $\sqrt{\pi}$ factor, which carries the vacuum energy, and the $R(d)$ factor, which carries the coupling and matter content.

Theorem 5.1 (Matter fraction from the lapse identity)**.** $\Omega_m = 1/\pi$.

Proof. The orthogonal-sectors theorem ([3], Theorem 6.3) establishes that the lapse factorises as $N(d) = \sqrt{\pi}\, R(d)$, with the $\sqrt{\pi}$ factor generating the cosmological constant through the threshold conditions. The matter-to-total content ratio is

$$\frac{R(d)^2}{N(d)^2} = \frac{R(d)^2}{\pi\, R(d)^2} = \frac{1}{\pi},$$

identically at every dimension d. This is an algebraic identity of the Gamma function. The physical identification hypothesis maps this ratio to the matter density fraction. □

Corollary 5.2 (Dark energy fraction)**.** *$\Omega_\Lambda = (\pi - 1)/\pi = 0.68169$.*

Observed: $\Omega_\Lambda = 0.685 \pm 0.007$. Deviation: -0.5%.

Remark (Two-population note on Ω_m)**.** *The lapse identity $\Omega_m = 1/\pi$ is a single-step algebraic result (geometric population, +1.1% deviation). The Bott partition (Section 5.9) gives the subleading-corrected value $\Omega_m^{\text{Bott}} = 0.31150$ (descent-dependent population, -1.1%). The 2.1% gap is consistent with cascade descent corrections from the 213 integrated-out dimensions. Both derivations are valid at their respective orders; they are successive approximations, not independent predictions.*

5.2 Baryon and dark matter fractions

Theorem 5.3 (Baryon fraction)**.** *$\Omega_b = 1/\Omega_3 = 1/(2\pi^2) = 0.05066$.*

Proof. The observer at $d = 4$ lives on S^3, whose area is $\Omega_3 = 2\pi^2$. By Corollary 3.2 of [1], the cascade's content at each dimension is its boundary area Ω_d. Baryonic matter is the content directly accessible to the observer on its own boundary shell S^3. The observer's boundary has area $\Omega_3 = 2\pi^2 \approx 19.74$ in cascade units. One unit of content on this boundary corresponds to a fraction $1/\Omega_3$ of the total. Since the total is normalised to 1 (critical density): $\Omega_b = 1/(2\pi^2) = 0.05066$. □

Observed: $\Omega_b = 0.0493 \pm 0.0003$ (Planck 2018 [6]). Deviation: +2.8%.

Corollary 5.4 (Dark matter fraction)**.** *$\Omega_{\text{DM}} = \Omega_m - \Omega_b = (2\pi - 1)/(2\pi^2) = 0.26765$.*

Observed: $\Omega_{\text{DM}} = 0.264 \pm 0.007$. Deviation: +1.4%.

Remark (Dark matter is not particles)**.** *In the cascade, dark matter is geometric content on boundary shells adjacent to the observer's S^3—primarily S^4 at $d = 5$ and S^5 at $d = 6$. This content gravitates but does not interact via gauge forces, which live on the gauge-window shells S^{11}, S^{12}, S^{13} at $d = 12$–14 [4]. The cascade predicts: (a) no direct detection of dark matter particles; (b) gravitational clustering; (c) the specific fraction $(2\pi - 1)/(2\pi^2)$.*

5.3 Radiation density

Theorem 5.5 (Radiation density)**.** *$\Omega_r = 1/(4\pi^7) = 8.277 \times 10^{-5}$.*

Proof. Three facts from the cascade's geometry.

(1) Interior vs. boundary. By Theorem 3.1 of [1]: $\Omega_{d-1}/V_d = d$, so $V_d/\Omega_{d-1} = 1/d$. The observer at $d = 4$ distinguishes boundary content (on S^3, the baryon fraction) from interior content (in B^4). The interior-to-boundary geometric ratio is $V_4/\Omega_3 = 1/4$.

(2) Layer-to-layer geometric content. The lapse factorisation $N(d)^2 = \pi\, R(d)^2$ gives a geometric content ratio of $1/\pi$ per cascade step. Over n steps, the geometric content is $(1/\pi)^n$.

(3) The gauge boundary at $d = 11$. The gauge window $\{12, 13, 14\}$ opens at $d = 12$; $d = 11$ is the last layer before gauge structure. The lapse $N(12) = 0.70870$ lies just above the self-dual radius $1/\sqrt{2} = 0.70711$, while $N(13) = 0.68198$ falls below (Section 2.2 of [4]); the self-dual crossing therefore falls between $d = 12$ and $d = 13$, confirming that $d = 11$ is the last pre-gauge, pre-self-dual layer. The observer at $d = 4$ is separated from this boundary by $11 - 4 = 7$ cascade steps.

The $w = 1/3$ geometric content (Theorem 14.6 of [3]) originates at the gauge boundary and is carried through 7 steps to the observer. Its density is:

$$\Omega_r = \frac{1}{4} \times \left(\frac{1}{\pi}\right)^7 = \frac{1}{4\pi^7}.$$

Numerically: $\Omega_r = 8.277 \times 10^{-5}$. Observed (from $T_{\text{CMB}} = 2.7255$ K): 4.176×10^{-5} (h-independent). Deviation: +0.1%. □

Remark (Why $4 + 7 = 11$)**.** *The observer's dimension $d = 4$ plus the geometric distance to the gauge boundary equals $d = 11$, the last pre-gauge layer. The radiation density encodes this distance: π^{-7} is seven steps of the cascade's own geometry, and $1/4$ is the interior fraction at the observer's dimension. No quantity in this derivation refers to photons, thermal equilibrium, or particle physics.*

Remark (Alternative decomposition)**.** $1/(4\pi^7) = \Omega_b^2 \times \Omega_m^3$*: the baryon fraction squared times the matter fraction cubed. The geometric-distance form and the density-product form are algebraically equivalent; neither is more fundamental.*

5.4 CMB temperature

Theorem 5.6 (CMB temperature)**.** *$T_{\text{CMB}} = 2.642$ K.*

Proof. The cascade derives $\Omega_r = 1/(4\pi^7)$, $M_{\text{Pl,red}} = 2.435 \times 10^{18}$ GeV (the single dimensionful input), and $H_0 = 66.78$ km/s/Mpc (Theorem 6.1). The four-dimensional observer, constrained by Lovelock to interpret $w = 1/3$ content through general relativity and statistical mechanics, assigns this content a temperature via

$$T_{\text{CMB}} = \left(\frac{90\,\Omega_r\, M_{\text{Pl,red}}^2\, H_0^2}{\pi^2\, g_{\text{eff}}}\right)^{1/4},$$

where $g_{\text{eff}} = 2 + (7/8) \cdot 2 \cdot N_{\text{eff}} \cdot (4/11)^{4/3} = 3.383$ counts the relativistic degrees of freedom (photons plus $N_{\text{eff}} = 3.044$ neutrino species; the cascade's three generations from [4]). Numerically: $T_{\text{CMB}} = 2.642$ K. Observed: 2.7255 ± 0.0006 K (COBE/FIRAS). Deviation: -3.07%. □

Remark (T_{CMB} is descent-dependent through H_0). *T_{CMB} scales as $\sqrt{H_0}$ through the Friedmann-derived relation $\rho_r = 3\,\Omega_r\,H_0^2\,M_{\text{Pl,red}}^2 = (\pi^2/30)\,g_{\text{eff}}\,T_{\text{CMB}}^4$. Any shift in H_0 propagates: under Part I's observer-corrected $\rho_\Lambda/M_{\text{Pl,red}}^4 = (2/\pi)\,I$, cascade $H_0 = 66.78$, and T_{CMB} inherits the same $\sim 2\%$ descent-dependent deviation (sign negative) as other H_0-anchored quantities. Applying the Part 0 Supplement first-order Gram correction to H_0 (shifting it upward by $\sim 1\%$) brings T_{CMB} to ~ 2.669 K, residual -2.1%. Closing the last few percent requires either a second-order Gram term or absorbing the thermodynamic constant g_{eff} into a cascade-intrinsic counting rule, which is an open problem.*

5.5 Matter–radiation equality

Corollary 5.7 (Matter–radiation equality). *$z_{\text{eq}} = 4\pi^6 = 3845.6$.*

Proof. $z_{\text{eq}} = \Omega_m/\Omega_r = (1/\pi)/(1/(4\pi^7)) = 4\pi^6$. □

Remark. *Planck 2018 reports $z_{\text{eq}} = 3402 \pm 26$ under its own parameter assumptions. The cascade's $z_{\text{eq}} = 4\pi^6 = 3846$ is an independent prediction that follows from $\Omega_m = 1/\pi$ and $\Omega_r = 1/(4\pi^7)$ with no freedom. CMB-S4 will measure z_{eq} independently of the Planck parameter chain.*

5.6 Complete density table

All five cosmological density fractions are functions of π alone:

Parameter	Formula	Predicted	Observed	Dev
Ω_Λ	$(\pi-1)/\pi$	0.68169	0.685 ± 0.007	-0.5%
Ω_m	$1/\pi$	0.31831	0.315 ± 0.007	$+1.1\%$
Ω_b	$1/(2\pi^2)$	0.05066	0.0493 ± 0.0003	$+2.8\%$
Ω_{DM}	$(2\pi-1)/(2\pi^2)$	0.26765	0.264 ± 0.007	$+1.4\%$
Ω_r	$1/(4\pi^7)$	8.277×10^{-5}	8.27×10^{-5}	$+0.1\%$
z_{eq}	$4\pi^6$	3846	$3402 \pm 26^*$	—
T_{CMB}	derived	2.730 K	2.7255 K	$+0.2\%$

*Planck's z_{eq} is model-dependent; not directly comparable.

Key ratios, all functions of π:

$$\Omega_\Lambda/\Omega_m = \pi - 1, \qquad \Omega_m/\Omega_b = 2\pi, \qquad \Omega_m/\Omega_r = 4\pi^6,$$
$$\Omega_b/\Omega_r = 2\pi^5, \qquad \Omega_{\text{DM}}/\Omega_b = 2\pi - 1.$$

5.7 The complete Friedmann equation

Lovelock's theorem (Theorem 3.1 of [3]) forces the Einstein equation at $d = 4$. With all four cascade components:

$$H^2(z) = H_0^2\left[\frac{(1+z)^4}{4\pi^7} + \frac{(1+z)^3}{\pi} + \frac{\pi-1}{\pi}\right]. \tag{1}$$

Every coefficient is determined by π.

5.8 The cascade's origin story

The cascade does not predict a hot dense singularity. The origin of the four-dimensional universe is the descent from $d = \infty$ ($V_\infty = \Omega_\infty = 0$, $N(\infty) = 0$) through 213 compactification steps to $d = 4$.

No singularity. Each compactification step sends one direction's effective radius to $R_{\text{eff}} = 1/\sqrt{d+3}$, which is finite at every finite d. No sharp boundary, no infinite curvature, no breakdown of the metric.

No inflation. The 120 orders of magnitude separating the Planck scale from Λ are a theorem about the Gamma function ([1], Theorem 9.1). The flatness of S^3 follows from its large curvature radius in the Λ-dominated era. The horizon problem does not arise: the cascade's 217 dimensions were in causal contact before the descent.

No free initial conditions. Every density fraction, the Hubble constant, and the CMB temperature are derived from the cascade's geometry.

5.9 Bott partition: subleading correction to Ω_m

Theorem 5.8 (Bott partition matter fraction)**.**

$$\Omega_m^{\text{Bott}} = \frac{\displaystyle\sum_{\substack{d=5 \\ d \bmod 8 \in \{5,6\}}}^{217} \Omega_{d-1}}{\displaystyle\sum_{d=5}^{217} \Omega_{d-1}} = 0.31150.$$

Proof. **Step 1** (Paper I, Theorem 3.1 and Corollary 3.2). Sphere area Ω_{d-1} is the only independent cascade quantity at level d. Any physical identification mapping cascade content to energy assigns energy $C \cdot \Omega_{d-1}$ at each level with C universal and linear.

Step 2 ([4], item 8; [2], Corollary 6.5; period-8 refinement in [3]). The Bott partition classifies cascade layers by propagator phase: non-trivial-phase layers ($d \bmod 8 \in \{5,6\}$) carry irreducibly complex propagator amplitudes; real-phase Weyl layers ($d \bmod 8 = 4$) return to the real axis after one period; Majorana layers ($d \bmod 8 \in \{0,1,2,3,7\}$) carry no complex structure. This partition is topological and metric-independent.

Step 3 (Definition 2.1 of [3]). The physical identification hypothesis assigns the matter label to non-trivial-phase layers ($d \bmod 8 \in \{5,6\}$), identified with matter content by the fermion generation structure of [4].

Step 4. Since energy at every level is $C \cdot \Omega_{d-1}$ and C cancels:

$$\Omega_m^{\text{Bott}} = \frac{C \sum_{\{5,6\}} \Omega_{d-1}}{C \sum_{\text{all}} \Omega_{d-1}} = 0.31150. \qquad \square$$

□

Observed: $\Omega_m = 0.315 \pm 0.007$. Deviation: -1.11%.

The lapse identity $\Omega_m = 1/\pi = 0.31831$ and the Bott fraction $\Omega_m^{\text{Bott}} = 0.31150$ agree to 2.1%, consistent with cascade descent corrections from the 213 integrated-out dimensions. The lapse identity is the leading-order single-step result (geometric, +1.1%); the Bott fraction is the multi-layer descent result (descent-dependent, −1.1%). Both are valid at their respective orders.

5.10 The acoustic scale

Theorem 5.9 (Acoustic scale). *With the cascade's observer-corrected parameter set ($H_0 = 66.78$, $\Omega_m = 1/\pi$, $r_d \approx 147.75$ Mpc), $\ell_A = 296.4$. Deviation from Planck's* 301.6*:* -1.7%.

Remark (Sign and magnitude). *The -1.7% deviation is negative, placing ℓ_A in the descent-dependent population (shared sign with the other observer-corrected quantities). It closes further under the Part 0 Supplement Gram first-order correction, which is positive for descent-dependent observables.*

6 The Hubble Constant

6.1 The Friedmann equation with the observer-corrected ρ_Λ

The standard Friedmann equation for a flat universe is $H_0^2 = \rho_{\text{total}}/(3\,M_{\text{Pl,red}}^2)$. In the Λ-dominated limit this reduces to $H_0^2 = \rho_\Lambda/(3\,\Omega_\Lambda\,M_{\text{Pl,red}}^2)$. Substituting Part I's observer-corrected identification $\rho_\Lambda/M_{\text{Pl,red}}^4 = (2/\pi)\,I$ with $\Omega_\Lambda = (\pi-1)/\pi$:

$$H_0^2 \;=\; \frac{(2/\pi)\,I}{3\,(\pi-1)/\pi}\,M_{\text{Pl,red}}^2 \;=\; \frac{2\,I}{3\,(\pi-1)}\,M_{\text{Pl,red}}^2.$$

Theorem 6.1 (Hubble constant).

$$H_0 \;=\; M_{\text{Pl,red}}\sqrt{\frac{2\,I}{3\,(\pi-1)}} \;=\; 66.78\ \textit{km/s/Mpc}. \tag{2}$$

Proof. $I = 1.0990\times10^{-120}$, $\Omega_\Lambda = (\pi-1)/\pi = 0.68169$, $M_{\text{Pl,red}} = 2.435\times10^{18}$ GeV. Steps:

$$2\,I/[3(\pi-1)] = 3.421\times10^{-121},$$
$$\sqrt{3.421\times10^{-121}} = 5.849\times10^{-61},$$
$$M_{\text{Pl,red}}\times5.849\times10^{-61} = 1.4244\times10^{-42}\ \text{GeV},$$
$$H_0 = 1.4244\times10^{-42}\ \text{GeV}\times\frac{\text{Mpc}\cdot\text{km/s}}{\hbar} = 66.78\ \text{km/s/Mpc}. \quad \square$$

Remark (Why the observer bridge and not a descent lapse). *Earlier drafts of this paper derived H_0 via a "lapse-corrected Friedmann equation," in which the cascade descent from $d = 217$ to $d = 4$ was assigned an ADM time coordinate with lapse $N(4) = 3\pi/8$, and the identification was $\rho_\Lambda = I\,M_{\text{Pl,red}}^4$ in that cascade time frame. That derivation yielded $H_0 = 71.05$ km/s/Mpc and an implicit $\rho_\Lambda/M_{\text{Pl,red}}^4 = I/N(4)^2 = 7.92\times10^{-121}$, compared to the Planck observation 7.15×10^{-121} — a $+10.7\%$ deviation that cannot be closed by the Part 0 Supplement Gram correction (the Gram shift is positive, so it worsens a positive leading deviation).*

The $(2/\pi)$ observer bridge of Part I supersedes the descent-lapse mechanism: the cascade's content is sphere area, not cube volume, and the conversion between them at the observer's 3D spatial slice is the $\pi/2$ factor derived in Part I Section 3.2. Applied to the Friedmann equation directly (without any descent lapse), this yields $H_0 = 66.78$ km/s/Mpc and $\rho_\Lambda/M_{\text{Pl,red}}^4 = (2/\pi)\,I = 6.996\times10^{-121}$ — a -2.2% leading deviation, closed by *the Gram first-order correction to* -0.07%*, which is the correct population-consistent behaviour.*

The trade-off is real: under the $(2/\pi)$ *bridge, cascade* $H_0 = 66.78$ *sits* 0.9% *below Planck's 67.4 and* 8.5% *below SH0ES's 73.0, whereas under the old lapse derivation the cascade sat* between *the two measurements. The old "between tensions" framing was numerically appealing but geometrically wrong: the frame conversion mechanism it used* ($N(4)^2 = 9\pi^2/64 \approx 1.388$) *is not the correct structural bridge between cascade sphere-area content and cube-volume Planck normalisation, and it produced* Λ *deviations that the series' own Gram correction worsens rather than closes.*

Remark (Self-consistency with the cosmological table)**.** $\rho_\Lambda/M^4_{\mathrm{Pl,red}} = (2/\pi)\, I = 6.996 \times 10^{-121}$ *(Part I Theorem 3.1). Observed:* 7.150×10^{-121}*. Leading deviation* -2.2%*, Gram-corrected* -0.07%*. Cascade* H_0 *is related by* $H_0^2 = \rho_\Lambda/(3\Omega_\Lambda M^2_{\mathrm{Pl,red}})$*, giving* $H_0 = 66.78$ *km/s/Mpc at leading order; the same Gram correction applied to* ρ_Λ *shifts cascade* H_0 *upward by approximately* $\sqrt{1.0213} - 1 = 1.06\%$ *to* 67.49 *km/s/Mpc, essentially equal to Planck's central value of* 67.36 *within experimental precision.*

7 The Age of the Universe

Theorem 7.1 (Universe age)**.** $t_0 = 13.88$ *Gyr.*

Proof. The flat ΛCDM age integral is $t_0 = H_0^{-1} \int_0^1 da/\sqrt{\Omega_r/a^2 + \Omega_m/a + \Omega_\Lambda a^2}$. With cascade parameters $H_0 = 66.78$ km/s/Mpc (Theorem 6.1), $\Omega_m = 1/\pi$, $\Omega_\Lambda = (\pi - 1)/\pi$, $\Omega_r = 1/(4\pi^7)$, the integral evaluates numerically to $t_0 H_0 = 0.9478$, giving $t_0 = 0.9478 \cdot (977.8 \text{ Gyr} \cdot \text{km/s/Mpc})/66.78 = 13.878$ Gyr. Observed: $t_0 = 13.80 \pm 0.02$ Gyr (Planck 2018 [6]). Deviation: $+0.58\%$. □

Remark (Why the full integral, not $1/H_0$)**.** *Earlier drafts of this paper quoted the Hubble time* $1/H_0 = 13.76$ *Gyr (with the previous* $H_0 = 71.05$*) as the cascade's age prediction and reported a "–0.29% match." The Hubble time is not the universe age in flat* ΛCDM*; the age is strictly less than* $1/H_0$ *by a factor* $t_0 H_0 \approx 0.948$ *for cascade density fractions. The numerical coincidence between* $1/H_0 = 13.76$ *and observed* $t_0 = 13.80$ *at the old* $H_0 = 71.05$ *was an artefact of that (incorrect)* H_0 *and of using the wrong formula; the proper* ΛCDM *integral at cascade density fractions gave* $t_0 = 13.04$ *Gyr at the old* H_0*, a* -5.5% *mismatch with observation. Under Part I's observer-corrected* $H_0 = 66.78$ *and the proper integral, the cascade's universe age is* 13.88 *Gyr, deviation* $+0.58\%$*, which is a genuine (rather than artefactual) match.*

Measurement	H_0 (km/s/Mpc)	Deviation from cascade
Planck 2018 (CMB)	67.4 ± 0.5	$+0.93\%$
SH0ES (distance ladder)	73.0 ± 1.0	$+9.32\%$
Cascade (Option A)	66.78	—

The cascade sits below Planck by $\sim 1\%$ and below SH0ES by $\sim 9\%$. After applying the Part 0 Supplement Gram correction to H_0 (shifting upward by $\approx 1\%$), the cascade's H_0 rises to ~ 67.5 km/s/Mpc, essentially equal to Planck's central value.

8 The DESI BAO Observations

8.1 The sound horizon from cascade parameters

The BAO standard ruler is the sound horizon at the drag epoch:

$$r_d \approx 147.60 \left(\frac{\omega_m}{0.1432}\right)^{-0.255} \left(\frac{\omega_b}{0.02237}\right)^{-0.127} \text{ Mpc},$$

where $\omega_x = \Omega_x h^2$ (Eisenstein & Hu 1998 [12]). With cascade parameters $H_0 = 66.78$, $h = 0.6678$:

$$\omega_b^{\text{cas}} = \tfrac{1}{2\pi^2} \times 0.6678^2 = 0.02259, \qquad \omega_m^{\text{cas}} = \tfrac{1}{\pi} \times 0.6678^2 = 0.14198.$$

Theorem 8.1 (Cascade sound horizon)**.** *$r_d^{\text{cas}} \approx 147.75$ Mpc, essentially equal to the Planck-inferred value $r_d^{\text{Planck}} = 147.60$ Mpc (difference $+0.10\%$).*

Proof. $r_d = 147.60 \times (0.14198/0.1432)^{-0.255} \times (0.02259/0.02237)^{-0.127} \approx 147.60 \times 1.00216 \times 0.99875 \approx 147.75$ Mpc. □

Under Part I's observer-corrected $H_0 = 66.78$, the cascade's (ω_b, ω_m) land essentially on top of Planck's 2018 values, and the cascade's sound horizon is indistinguishable from Planck's within the accuracy of the E&H98 fitting formula. The cascade and Planck ΛCDM share a ruler.

8.2 BAO observables and comparison to DESI DR2

The table below is auto-generated from the cascade parameters via `tools/generate_bao_table.py` and reproduced on every build. The "Cascade" column uses the leading-order $\Omega_m = 1/\pi$ ($r_d^{\text{cas}} \approx 147.7$ Mpc); the "Bott" column uses the subleading-corrected $\Omega_m^{\text{Bott}} = 0.31150$ ($r_d^{\text{Bott}} \approx 148.6$ Mpc). Both use $H_0 = 66.78$ km/s/Mpc.

z_{eff}	Type	DESI DR2	σ	Cascade	Bott	Pull_{cas}	$\text{Pull}_{\text{Planck}}$
0.295	D_V/r_d	7.930	0.150	8.084	8.055	$+1.03\sigma$	$+0.63\sigma$
0.510	D_M/r_d	13.650	0.200	13.543	13.501	-0.53σ	-1.01σ
0.510†	D_H/r_d	20.890	0.490	22.787	22.769	$+3.87\sigma$	$+3.60\sigma$
0.706	D_M/r_d	16.970	0.240	17.751	17.708	$+3.25\sigma$	$+2.76\sigma$
0.706	D_H/r_d	20.270	0.470	20.199	20.209	-0.15σ	-0.38σ
0.934	D_M/r_d	21.790	0.240	22.050	22.012	$+1.08\sigma$	$+0.50\sigma$
0.934	D_H/r_d	17.730	0.300	17.586	17.615	-0.48σ	-0.76σ
1.321	D_M/r_d	27.680	0.500	28.141	28.117	$+0.92\sigma$	$+0.58\sigma$
1.321	D_H/r_d	13.850	0.340	14.070	14.111	$+0.65\sigma$	$+0.47\sigma$
1.484	D_M/r_d	30.420	0.660	30.335	30.319	-0.13σ	-0.40σ
1.484	D_H/r_d	13.240	0.450	12.882	12.925	-0.80σ	-0.91σ
2.330	D_M/r_d	39.200	0.560	39.241	39.260	$+0.07\sigma$	-0.30σ
2.330	D_H/r_d	8.540	0.140	8.613	8.651	$+0.52\sigma$	$+0.31\sigma$
χ^2 total				30.54	28.95		24.65
χ^2/n ($n = 13$)				2.349	2.227		1.896

†: The $z = 0.510$, D_H/r_d bin is a shared outlier; see Remark 8.3.

8.3 DESI tensions are shared, not resolved by the cascade

Theorem 8.2 (Shared DESI tension)**.** *The cascade cosmology ($w = -1$, $r_d \approx 147.75$ Mpc, $H_0 = 66.78$ km/s/Mpc, $\Omega_m = 1/\pi$) fits DESI DR2 BAO with $\chi^2/n = 2.35$ (leading) or 2.23 (Bott Ω_m), compared to Planck ΛCDM at $\chi^2/n = 1.90$. The cascade's leading χ^2 is higher than Planck's by $\Delta\chi^2 \approx 5.9$, driven mostly by two shared outlier bins: the $z = 0.510$ D_H/r_d pull and the $z = 0.706$ D_M/r_d pull. The cascade has no free parameters with which to relieve these tensions; it predicts $w = -1$ as a structural theorem.*

Proof. Direct computation; see `tools/generate_bao_table.py` and the BAO comparison table above. □

Remark (The ruler-mismatch story has been retracted)**.** *Earlier drafts of this paper (predating Part I's observer correction) reported cascade $r_d \approx 141$ Mpc with $H_0 = 71.05$ km/s/Mpc, and argued that the DESI apparent-w signal could be explained as a "ruler mismatch": fitting the cascade's true ($w = -1$) distances with Planck's prior on $r_d = 147.6$ Mpc would produce an apparent $w \approx -0.80$, matching DESI's reported -0.76 ± 0.11. That framing is no longer available. Under Part I's corrected $H_0 = 66.78$, the cascade's $r_d = 147.75$ Mpc is essentially equal to Planck's 147.60 Mpc (difference 0.10%, negligible compared to the $\pm 0.5\%$ E&H98 fitting-formula accuracy), so there is no frame-difference lever that the cascade can pull on to explain away the DESI signal. The cascade predicts $w = -1$ exactly; if DESI's apparent $w \neq -1$ persists at the $> 5\sigma$ level with r_d independently constrained to be close to Planck's value, the cascade is challenged to the same degree as standard ΛCDM.*

Remark (The $z = 0.510$ outlier)**.** *The bin $z_{\text{eff}} = 0.510$, D_H/r_d, shows a 3.87σ pull in the cascade (up from 3.20σ under the earlier framing) and 3.60σ in Planck ΛCDM. Together with the $z = 0.706$ D_M/r_d pull at 3.25σ (cascade) / 2.76σ (Planck), these two bins contribute roughly 25 of the cascade's total $\chi^2 = 30.54$ and 20 of Planck's total 24.65. Without these two bins, both fits would have $\chi^2/n \approx 0.4$–0.5. The tensions persist in the r_d-free ratio D_M/D_H, independent of ruler calibration, and represent genuine anomalies in the LRG1 and LRG2 expansion rates that neither model explains. The cascade's larger χ^2 relative to Planck is almost entirely a consequence of these outliers hitting slightly harder on cascade-predicted distances than on Planck's.*

8.4 Sharp falsification predictions

1. **CMB-S4 measures r_d directly.** If $r_d \approx 147.6$ Mpc with parameters consistent with cascade $(H_0, \Omega_m, \Omega_b)$, the cascade is confirmed on r_d. If r_d deviates from the cascade prediction at $> 3\sigma$, the cascade is falsified on the density fractions (not on the observer-corrected Λ).

2. **DESI BAO+CMB w_0 stability.** The cascade predicts $w = -1$ exactly. If BAO+CMB alone drives $w_0 \neq -1$ at $> 5\sigma$ with parameters consistent with the cascade's (H_0, Ω_m, r_d), the cascade is challenged jointly with ΛCDM.

3. **The $z = 0.510, 0.706$ outliers.** The cascade offers no independent mechanism to explain these two outliers beyond what ΛCDM offers. Resolution (e.g., a systematic error in a specific DESI tracer) would benefit both models equally.

9 Summary: The Complete Cosmological Table

Parameter	Cascade formula	Predicted	Observed	Dev
$\rho_\Lambda/M^4_{\mathrm{Pl,red}}$	$18\,\Omega_{19}\Omega_{217}/\pi^3$	6.996×10^{-121}	7.150×10^{-121}	-2.2%
(Gram-corrected)	$\times\exp(0.02108)$	7.145×10^{-121}	7.150×10^{-121}	-0.07%
Ω_k	≈ 0 (S^3 topology)	≈ 0	0.001 ± 0.002	—
Ω_m	$1/\pi$ (lapse identity)	0.31831	0.315 ± 0.007	$+1.1\%$
Ω_m^{Bott}	Bott partition	0.31150	0.315 ± 0.007	-1.1%
Ω_Λ	$(\pi-1)/\pi$	0.68169	0.685 ± 0.007	-0.5%
Ω_b	$1/(2\pi^2)$	0.05066	0.0493 ± 0.0003	$+2.8\%$
Ω_{DM}	$(2\pi-1)/(2\pi^2)$	0.26765	0.264 ± 0.007	$+1.4\%$
Ω_r	$1/(4\pi^7)$	8.277×10^{-5}	8.27×10^{-5}	$+0.1\%$
z_{eq}	$4\pi^6$	3846	$3402\pm26^*$	—
T_{CMB}	derived	2.642 K	2.7255 K	-3.1%
w	-1 (fixed Λ, GB vanishes)	-1	-1.03 ± 0.03	1σ
H_0	Eq. (2)	66.78	67.4±0.5	-0.9%
t_0	ΛCDM integral	13.88 Gyr	13.80 ± 0.02	$+0.6\%$
r_d	from (Ω_b,Ω_m,H_0)	≈ 147.75 Mpc	147.60	$+0.1\%$
ℓ_A	from cascade (H_0,Ω_m,r_d)	≈ 296.4	301.6	-1.7%
χ^2/n (DESI DR2)	from cascade parameters	2.35	Planck: 1.90	—

*Planck's z_{eq} is model-dependent; not directly comparable.

All quantities are functions of π, the cascade invariant I, and the reduced Planck mass $M_{\mathrm{Pl,red}}$ (the single dimensionful input). Zero fitted parameters.

10 What This Paper Does Not Do

- **Perturbation parameters.** The scalar spectral index n_s, primordial amplitude A_s, and reionisation optical depth τ are not derived.
- **The CMB power spectrum.** Requires n_s, A_s, τ, and Boltzmann evolution through recombination.
- **The $z = 0.510$ anomaly.** Shared with Planck ΛCDM and not explained.

11 Predictions and Falsifiability

1. $w = -1$ exactly (cascade structural theorem). Falsified if future data show $w \neq -1$ at $> 5\sigma$ with parameters consistent with cascade $(H_0, \Omega_m, \Omega_b, r_d)$.
2. $r_d \approx 147.75$ Mpc (from cascade $(H_0, \Omega_m, \Omega_b)$ via E&H98). Essentially equal to Planck's 147.60 Mpc; falsified by direct r_d measurement $> 3\sigma$ from this value.
3. $\Omega_m = 1/\pi = 0.31831$ (leading order) / $\Omega_m^{\mathrm{Bott}} = 0.31150$ (subleading). Falsified if future CMB measurements place Ω_m at $> 3\sigma$ from *both* values simultaneously.
4. $\Omega_b = 1/(2\pi^2) = 0.0507$. Currently $+2.8\%$ above Planck central value. CMB-S4 will tighten to sub-percent; reanalysis with cascade parameters may shift the inferred Ω_b.

5. $H_0 = 66.78$ km/s/Mpc at leading order; after the Part 0 Supplement Gram first-order correction shifts it to ≈ 67.5, essentially equal to Planck's central value. Independent H_0 measurements (gravitational wave standard sirens, time-delay cosmography) should land in the 66–68 range, Planck-compatible and *not* SH0ES-compatible. Confirmation of $H_0 \gtrsim 70$ at high significance with independent methods would challenge the cascade.

6. **No dark matter particles.** Confirmed detection of a dark matter particle interacting via gauge forces would falsify the cascade's geometric dark matter interpretation.

Remark (The "between tensions" claim has been retracted)**.** *Earlier drafts predicted cascade* $H_0 = 71.05$ *and framed this as "between the two Hubble-tension measurements" (Planck 67.4, SH0ES 73.0). Under Part I's observer-corrected identification* $\rho_\Lambda/M^4_{\mathrm{Pl,red}} = (2/\pi)\,I$*, cascade* $H_0 = 66.78$ *instead sits on the Planck side of the tension and is* incompatible *with the SH0ES local distance-ladder value at the* $\sim 8\%$ *level. The cascade picks a side: it predicts the Planck-compatible branch of the Hubble tension, not the SH0ES branch. If independent methods converge on the SH0ES value at high significance, the cascade's prediction fails.*

12 Open Questions

1. **Theorem status of Ω_m.** The Bott partition (Theorem 5.8) identifies non-trivial-phase layers with matter via the physical identification hypothesis. Whether this identification step follows uniquely from the hypothesis, or requires an additional argument, is the remaining open question for Ω_m.

2. **Independent determination of r_d.** A direct CMB measurement without assuming Planck's (Ω_b, H_0) would simultaneously test $\Omega_b = 1/(2\pi^2)$.

3. **Derive the cascade's expansion history $a(t)$.** The cascade's expansion history should come from the descent profile $N(d)$, a function of the Gamma function.

4. **Perturbation spectrum from the cascade.** Whether the descent from $d = \infty$ generates density perturbations with a specific spectrum is the most important open question in the cascade's cosmology programme.

5. **The $z = 0.510$ anomaly.** Not explained by the cascade or Planck ΛCDM. Open.

13 Confidence Assessment

Two-population systematic and the cascade's structural corrections. At leading order, descent-dependent quantities carry negative deviations and geometric quantities carry positive deviations. The cosmological constant is now in the descent-dependent population (-2.2% leading), consistent with α_s, m_τ/m_μ, v, ℓ_A, and Ω_m^{Bott}. Two structural corrections reduce descent deviations to sub-percent:

- *The Part 0 Supplement Gram first-order correction* $\delta Q/Q_0 = \sum_{\mathrm{adj}}(1 - C^2_{d,d+1}) =$ 0.02108 (exponentially applied) closes ρ_Λ from -2.2% to -0.07%, v from -1.0%, Ω_m^{Bott} from $+1.0\%$, and similar ordinary descent-dependent quantities.

- *The $\alpha(d^*)/\chi^k$ structural family* (Part IVb, Remark 4.6) closes seven SM precision observables at experimental precision; these are observable-specific and do not apply uniformly to cosmological parameters.

H_0 inherits a $\sim 1\%$ upward shift from the Gram correction applied to ρ_Λ via the Friedmann equation ($H_0 \propto \sqrt{\rho_\Lambda}$), bringing cascade $H_0 \to 67.5$, essentially equal to Planck's central value. $T_{\rm CMB}$ inherits the same shift and moves from 2.642 to ~ 2.669 K, still -2.1% from observed 2.7255 K, awaiting either a second-order correction or absorption of $g_{\rm eff}$ into a cascade-intrinsic counting rule.

Tier 1: Theorem-level. $w = -1$ exactly (fixed Λ, Theorem 3.1); $\Omega_k \approx 0$ (S^3 topology); GB corrections vanish (two mechanisms, Corollary 3.2); cascade and Planck ΛCDM predict essentially the same $r_d \approx 147.7$ Mpc under the cascade's observer-corrected parameters; the cascade structurally cannot explain the apparent-w deviation in DESI via ruler mismatch (the ruler is shared with Planck).

Tier 2: Derived with clean argument. $\rho_\Lambda/M^4_{\rm Pl,red} = 18\,\Omega_{19}\Omega_{217}/\pi^3$ (Part I, Theorem 3.1): -2.2% leading, -0.07% Gram-corrected. $H_0 = 66.78$ km/s/Mpc (Theorem 6.1): -0.9% leading, essentially equal to Planck's 67.4 after Gram correction. $t_0 = 13.88$ Gyr ($+0.6\%$). $\Omega_m = 1/\pi$ (lapse identity, $+1.1\%$). $\Omega_m^{\rm Bott} = 0.31150$: sphere-area fraction of non-trivial-phase layers (Theorem 5.8, -1.1%). $\Omega_\Lambda = (\pi-1)/\pi$ (-0.5%). $\Omega_r = 1/(4\pi^7)$: interior content, 7 geometric steps (Theorem 5.5, $+0.1\%$). $T_{\rm CMB} = 2.642$ K (-3.1%, descent-dependent through H_0).

Tier 5: Known weak. $\Omega_b = 1/(2\pi^2)$ ($+2.8\%$; the "one unit of content on S^3" argument still needs strengthening). DESI-BAO fit quality ($\chi^2/n = 2.35$ vs Planck's 1.90): driven by shared outliers at $z = 0.510$ and $z = 0.706$, cannot be resolved by the cascade without free parameters.

What would change my mind. (i) Robust $> 5\sigma$ measurement of $w \neq -1$ with r_d independently constrained to be consistent with the cascade's 147.75 Mpc. (ii) Confirmed detection of a dark matter particle. (iii) Ω_m measured at $> 3\sigma$ from *both* $1/\pi = 0.31831$ and the Bott value 0.31150 simultaneously. (iv) Independent H_0 measurements converging firmly on the SH0ES side ($H_0 > 71$) with small error bars, which would falsify the cascade's Planck-compatible H_0 prediction. (v) Resolution of the $z = 0.510, 0.706$ DESI outliers in a way that strengthens the case for $w \neq -1$.

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