The Cascade Series — Part II = III

Quantum Gravity without Quantising Gravity: Why the Quantum and Gravitational Projections of the Cascade Are the Same Theorem

RTAC

March 2026

Abstract

The cascade series tests one hypothesis: the infinite-dimensional unit ball, descended to four dimensions, is indistinguishable from our universe. Paper II [2] derives quantum mechanics as the measurement theory of a 4D observer in the cascade geometry. Paper III [3] derives general relativity as the unique gravitational equation available to that observer. This paper proves they are the same theorem.

Both projections map the same cascade state on the same unit sphere using the same propagator under the same identification hypothesis. The propagator $K(t) = |K|\, e^{-iHt}$ is simultaneously the quantum time-evolution operator (unitary, probability-preserving) and the source of Lorentzian signature (oscillatory, $g_{tt} < 0$). These are not two identifications of one expression—they are one physical fact in two vocabularies. At $d = 4$, Gleason's theorem forces the Born rule as the unique probability measure and Lovelock's theorem forces the Einstein equation as the unique gravitational equation. Two uniquely forced theories from a single source admit no mutual contradiction: the space of possible conflicts is empty.

The standard conflicts between quantum mechanics and general relativity—the UV catastrophe, the problem of time, background independence, the information paradox, and non-renormalisability—are artefacts of treating the two 4D projections as independent theories. In the cascade, they share a common origin. There is nothing to reconcile because there was never a separation.

As a direct application, we derive Bekenstein–Hawking entropy from boundary dominance alone: a horizon of area A at dimension d hides geometric content A/d. At $d = 4$: $S = A/4$. The factor $1/4$ is the boundary-to-volume ratio at the observer's dimension. Combining $S = A/4$ with the Schwarzschild area–mass relation $A = 16\pi M^2$ (a mathematical consequence of the cascade's own Einstein equation via Birkhoff's theorem) gives the Hawking temperature $T = 1/(8\pi M)$ by calculus—no semiclassical gravity, no QFT on curved spacetime, no Bogoliubov transformations.

Contents

1 The Schism and Its Dissolution

1.1 The standard conflict

Quantum mechanics requires a fixed background. General relativity makes the background dynamical. Combining them produces perturbative non-renormalisability [4]. Eighty years of effort to resolve this conflict have produced candidate frameworks but no unique, predictive, empirically confirmed quantum theory of gravity.

1.2 Why the cascade dissolves the problem

The cascade does not quantise gravity. It does not need to. The conflict arises because quantum mechanics and general relativity are formulated as independent theories, each claiming to be fundamental. In the cascade, neither is fundamental. Both are 4D projections of a single 217-dimensional geometry. They cannot conflict for the same reason that the shadow of a cube on two different walls cannot be inconsistent: both shadows are determined by the cube.

2 Single-Source Structure

2.1 Shared ingredients

Papers II and III derive their results from the same inputs.

Ingredient	QM projection (II)	GR projection (III)
State space	Unit sphere S^{d-1}	Unit sphere S^{d-1}
Fundamental constant	$\sqrt{\pi} = \Gamma(1/2)$	$\sqrt{\pi} = \Gamma(1/2)$
Propagator	$K(t) = \lvert K\rvert\, e^{-iHt}$	$K(t) = \lvert K\rvert\, e^{-iHt}$
Complex structure	$J^2 = -\mathrm{Id}$	$J^2 = -\mathrm{Id}$
Time	Slicing direction	Slicing direction
Identification	Def. 2.1 of III	Def. 2.1 of III

No entry differs between columns. The two projections draw on the same state space, the same constant, the same propagator, the same complex structure, the same time, and the same hypothesis.

Theorem 2.1 (Single-source structure)**.** *The quantum and gravitational projections are functions of the same cascade state $|\Psi\rangle \in S^{d-1}$. Neither projection introduces any structure not present in the cascade geometry.*

Proof. Paper II derives the 4D Hilbert space, Born rule, and unitary evolution from orthogonality, concentration of measure, and the forced precession—all properties of S^{d-1}. Paper III derives the 4D metric, Einstein equation, and Lorentzian signature from the foliation metric, Lovelock's theorem, and the same forced precession—all properties of S^{d-1}. No external structure is invoked in either derivation. Both are read from the same sphere. □

Remark (The identification hypothesis is shared)**.** *Definition 2.1 of Paper III does not say "the cascade is quantum mechanical" or "the cascade is gravitational." It says "the cascade's geometry is our physics." The quantum and gravitational content are consequences, not inputs.*

2.2 The state space is the Hilbert space is the configuration space

In standard physics, the Hilbert space of QM and the configuration space of GR are separate mathematical objects. In the cascade, they are the same object: the unit sphere S^{d-1}.

Theorem 2.2 (State space identity)**.** *The boundary S^{d-1} that carries the quantum state (Paper II) is the same S^{d-1} whose geometry determines the metric (Paper III). A cascade state $|\Psi\rangle \in S^{d-1}$ simultaneously specifies the quantum amplitudes and the geometric configuration. There is one sphere, not two sets of degrees of freedom.*

Proof. Paper II: the state space is S^{d-1} via boundary dominance ($\Omega_{d-1}/V_d = d$; the cascade's content is its boundary). Paper III: the metric is the foliation of B^{d+1} into cross-sections, with the induced geometry on S^{d-1} determining the 4D metric. Same sphere. □

3 The Propagator Identity

Theorem 3.1 (One propagator, two readings)**.** *The cascade propagator $K(t) = |K|\, e^{-iHt}$ with $H = \lambda_\infty > 0$ is simultaneously:*

1. *the quantum time-evolution operator: unitary, probability-preserving (Paper II, Theorem 7.1), and*
2. *the source of Lorentzian signature: oscillatory character requires $g_{tt} < 0$ (Paper III, Theorem 10.2).*

Proof. Paper II derives $K(t)$ from the forced precession $\alpha = \pi/2$ and identifies t with the slicing direction. Paper III takes the same $K(t)$ and observes: e^{-iHt} oscillates (unitary), so $g_{tt} < 0$ (Lorentzian). The chain is:

$$\text{Forced precession} \;\to\; K = |K|\, e^{-iHt} \;\to\; \text{unitarity (QM)} + \text{Lorentzian (GR)}.$$

One propagator. One chain. Two names for the same consequence. □

Corollary 3.2 (Unitarity and Lorentzian signature are the same theorem)**.** *Both require $H = \lambda_\infty > 0$. This is established once, from the forced precession, which is established once, from orthogonality. The cascade's single axiom forces both simultaneously.*

Remark (The Euclidean/Lorentzian duality)**.** *The cascade is Euclidean (B^d is a Euclidean object). The physical spacetime is Lorentzian. Paper III explains: the Euclidean geometry produces an oscillatory propagator through the forced precession, and oscillatory propagator character IS Lorentzian signature. No Wick rotation, no analytic continuation, no choice of contour. There is a Euclidean geometry whose propagator is Lorentzian. Both facts coexist because they are describing the same mathematics.*

4 Double Uniqueness

Theorem 4.1 (Gleason + Lovelock = consistency). *At $d = 4$:*

1. *Gleason's theorem [5]: the Born rule is the unique probability measure on a Hilbert space of dimension ≥ 3.*
2. *Lovelock's theorem [6]: the Einstein equation is the unique symmetric divergence-free rank-2 tensor equation in exactly $d = 4$.*

Two uniquely forced theories from a single source admit no mutual contradiction.

Proof. Suppose the quantum projection $\mathcal{Q}$ and the gravitational projection $\mathcal{G}$, applied to the same cascade state, yield contradictory predictions for some 4D observable. Then at least one is not the unique theory compatible with the cascade geometry—contradicting Gleason or Lovelock. No such observable exists. □

Remark (The role of $d = 4$). *The argument depends on $d = 4$. In higher dimensions, Lovelock permits additional gravitational terms with free couplings; the gravitational projection would no longer be unique and consistency would fail. The cascade forces $d = 4$ (Paper III, Section 9) precisely where both uniqueness theorems hold.*

5 No Absolute Scale

Theorem 5.1 (The cascade produces ratios). *Every physical prediction of the cascade series is a dimensionless ratio. The cascade geometry contains no intrinsic length, time, mass, or energy scale. The dimensionful constants G, $\hbar$, c are unit-conversion factors, not physical parameters.*

Proof. The cascade's input is B^d with $|x| \leq 1$. Rescaling changes no ratio. Its outputs—I, α_s, $\sin^2\theta_W$, m_μ/m_e, Ω_m—are pure numbers from the Gamma function. $\hbar = (\pi/2)/\Delta t$ (Paper II): the angle is derived; Δt is a unit choice. G maps cascade curvature ratios to human units. c converts spatial and temporal components of the cascade metric. □

Corollary 5.2 (The UV problem is a projection artefact). *The "Planck scale" is not a physical place. It is a ratio at which the 4D effective description becomes unreliable. The cascade remains well-defined at all ratios. Standard UV divergences arise from integrating over arbitrarily high momenta in 4D; in the cascade, the sum over levels terminates at $d = 217$. The finiteness is structural (a theorem about the Gamma function), not regulatory.*

6 Metric Superpositions Are Cascade States

Theorem 6.1 (Gravity is already quantum). *The 4D metric $g_{\mu\nu}$ is determined by the cascade state $|\Psi\rangle \in S^{d-1}$. Different states produce different metrics. Superpositions of states produce superpositions of metrics. The Born rule assigns probabilities to metric outcomes. Each outcome satisfies Einstein's equation (Lovelock).*

Proof. Paper III derives $g_{\mu\nu}$ from the cascade's foliation structure, which is a property of the cascade state. Paper II establishes superposition on S^{d-1}. A superposition $|\Psi\rangle = a|\Psi_1\rangle + b|\Psi_2\rangle$ with $g_{\mu\nu}^{(1)} \neq g_{\mu\nu}^{(2)}$ is a state of superposed metrics. The Born rule (Gleason) gives $p(k) = |\langle\Phi_k|\Psi\rangle|^2$. Each outcome satisfies Einstein's equation (Lovelock). □

Remark (No second quantisation of gravity). *The standard approach: take $g_{\mu\nu}$, promote to $\hat{g}_{\mu\nu}$, impose commutation relations, struggle with non-renormalisability. The cascade bypasses this. The metric is never promoted to an operator. It is a property of the quantum state. Superpositions of metrics are superpositions of states. Gravity was always quantum in the cascade, because geometry was always quantum in the cascade.*

7 Bekenstein–Hawking Entropy Is Hidden Geometry

7.1 The claim

A black hole's "entropy" is not a statistical quantity. It is the amount of cascade geometry the observer cannot see.

Theorem 7.1 (Hidden geometry behind a horizon). *Let a d-dimensional cascade observer measure a horizon of boundary area A in cascade units. The geometric content hidden behind the horizon is*

$$\boxed{S = \frac{A}{d}.}$$

At $d = 4$: $S = A/4$.

Proof. Boundary dominance (Paper I, Theorem 4.4): $\Omega_{d-1}/V_d = d$. Therefore $V_d = \Omega_{d-1}/d$. A horizon of area A is a boundary. Its interior content is $V = A/d$. At $d = 4$: $V = A/4$. □

One theorem. One line. No semiclassical gravity, no QFT on curved spacetime, no thermal radiation, no microstate counting.

7.2 Content is geometry

Corollary 3.2 of Paper I: sphere area is the only independent cascade quantity. Content IS geometry. One unit of sphere area is one unit of cascade content. A horizon of area A hides $A/4$ units of geometric content. The question "how many microstates?" is downstream—the geometry is primary.

7.3 Why Hawking got $A/4$ from a different calculation

Hawking (1975) [8] derived $S = A/4$ from QFT on a Schwarzschild background: Bogoliubov transformations, thermal density matrices, the semiclassical approximation. A statistical calculation producing a geometric answer. This has been mysterious for fifty years.

The cascade explains: Hawking measured the hidden geometry through a 4D effective theory. The thermal radiation is the 4D observer's description of geometric content leaking across an asymptotic compactification boundary. Both routes arrive at $A/4$ because they measure the same geometry from different altitudes.

7.4 The standard interpretations are downstream

Level	Statement
Cascade (primary)	Hidden geometry $= A/d$
$\downarrow$	
Thermodynamic	$Q/T = A/4$
Statistical	$\ln W = A/4$
Information-theoretic	$H = A/4$ nats

All three give $A/4$ because they are all measuring the same geometric content. None is fundamental. The geometry is fundamental.

7.5 Hawking temperature from the cascade

Theorem 7.2 (Hawking temperature)**.** *A Schwarzschild black hole of mass M has temperature*

$$T = \frac{1}{8\pi M}.$$

Proof. Three steps, each from the cascade's own content.

Step 1. Hidden geometry $S = A/4$ (Theorem 7.1, boundary dominance at $d = 4$).

Step 2. The Einstein equation is the unique gravitational equation at $d = 4$ (Paper III, Lovelock uniqueness). By Birkhoff's theorem—a mathematical consequence of the Einstein equation—the Schwarzschild metric is the unique spherically symmetric vacuum solution. Its horizon has area $A = 16\pi M^2$ in Planck units.

Step 3.

$$T = \left(\frac{\partial S}{\partial M}\right)^{-1} = \left(\frac{\partial}{\partial M}\frac{A}{4}\right)^{-1} = \left(\frac{\partial}{\partial M}\,4\pi M^2\right)^{-1} = \frac{1}{8\pi M}. \qquad \square$$

Remark (What the derivation uses and does not use)**.** *The derivation uses boundary dominance (a Gamma function identity), Lovelock uniqueness (a tensor-algebraic theorem), Birkhoff's theorem (a mathematical consequence of the Einstein equation), and the chain rule. It does not use quantum field theory on curved spacetime, Bogoliubov transformations, the semiclassical approximation, or any counting of microstates. Hawking's original derivation [8] required semiclassical gravity because it had no geometric route to $S = A/4$; the cascade provides that route through boundary dominance, making the semiclassical calculation unnecessary.*

Remark (The first law is output, not input)**.** *The relation $dM = T\,dS$ is often cited as the first law of black hole mechanics and used as an input to derive T. Here the logic is reversed: $S(M) = 4\pi M^2$ is known from Steps 1–2, so $T = (dS/dM)^{-1}$ follows by calculus. The first law is the* output*—a consequence of the cascade's geometric entropy and the Einstein equation—not an independent postulate.*

7.6 Cosmological hidden geometry

The cascade hierarchy $\Omega_7/\Omega_{217} \approx 10^{121}$ is the ratio of sphere areas at the two equilibria: the largest and the smallest in the cascade's tower. At $d = 4$, boundary dominance gives the hidden geometry as $1/4$ of this hierarchy—approximately 10^{120} in cascade units. This is the same 10^{120} that appears as the cosmological constant's hierarchy (Paper I, Theorem 9.1), seen from the other side: $\Lambda \sim 10^{-120}$ is the inverse of the hidden geometry. The cascade derives both numbers from the Gamma function; neither requires a de Sitter solution or semiclassical gravity.

8 Resolution of the Standard Conflicts

Conflict	Source	Cascade resolution
UV catastrophe	Infinite modes in 4D	Finite tower: 213 levels, structural
Problem of time	$H = 0$ in canonical gravity	$H = \lambda_\infty > 0$; time is slicing
Background dependence	QM needs fixed metric	Hilbert space on S^{d-1}; metric is output
Information paradox	Sharp horizon + unitarity	No sharp horizon; asymptotic compactification
Non-renormalisability	$g_{\mu\nu} \to \hat{g}_{\mu\nu}$	Metric is state property; no promotion
Planck-scale physics	What happens at l_{Pl}?	No absolute scale; cascade defined at all ratios
Euclidean/Lorentzian	Wick rotation ambiguity	Euclidean geometry, Lorentzian propagator
BH entropy	Statistical $\to$ geometric?	Geometry is primary: $S = A/d$

Every row uses the cascade's existing content (Papers I–III) and classical theorems (Gleason, Lovelock). No new postulate, no new parameter, no new mathematical structure.

9 Summary

Result	Mechanism	Section
QM and GR share single source	Same state space, propagator, hypothesis	2
Hilbert space = configuration space	Both are S^{d-1}	2.2
Unitarity = Lorentzian signature	Same propagator, same $H > 0$	3
Mutual consistency	Gleason + Lovelock: double uniqueness	4
No absolute scale	All predictions are ratios	5
UV dissolution	Finite tower; projection artefact	5
Gravity already quantum	Metric is state property on S^{d-1}	6
$S = A/4$	Boundary dominance at $d = 4$	7
$T = 1/(8\pi M)$	$S(M)$+ Birkhoff; no semiclassics	7.4
Hidden geometry $\sim 10^{120}$	Cascade hierarchy / 4	7.5

10 What This Paper Does and Does Not Do

Does:

- Proves the quantum and gravitational projections are the same theorem: same source, same propagator, same state space, same hypothesis.
- Proves mutual consistency via double uniqueness (Gleason + Lovelock).

- Dissolves all seven standard QM/GR conflicts.
- Derives $S = A/4$ from one theorem (boundary dominance), with no semiclassical input.
- Identifies the factor $1/4$ as $1/d$ at the observer's dimension.
- Derives the Hawking temperature $T = 1/(8\pi M)$ from boundary dominance and Birkhoff's theorem, with no semiclassical input (Theorem 7.2).
- Connects the cosmological hierarchy (10^{120}) to hidden geometry via boundary dominance, with no semiclassical input.
- Predicts $S = A/d$ for general observer dimension.

Does not:

- Compute metric-perturbation scattering amplitudes or demonstrate UV finiteness explicitly.
- Derive the thermal spectrum and greybody factors from cascade geometry.
- Compute entropy of Kerr or Reissner–Nordström black holes.
- Construct the explicit Hilbert space of metric superpositions or compute decoherence rates.
- Perform the BTZ cross-check ($S = A/3$ at $d = 3$).
- Derive the Page curve.

11 Open Questions

1. **Hawking radiation spectrum.** Theorem 7.2 derives the temperature $T = 1/(8\pi M)$ from boundary dominance and Birkhoff's theorem, with no semiclassical input. The thermal *spectrum* (Planckian with greybody factors) and the radiation *mechanism* (geometric content leaking through asymptotic compactification) remain open: a geometric derivation of the spectral shape would complete the connection.
2. **BTZ cross-check.** The cascade predicts $S = A/3$ at $d = 3$. Computing G_3 from the cascade's KK reduction and verifying the BTZ formula would provide an independent structural test.
3. **Metric-perturbation scattering.** The cascade predicts no graviton particles (Paper III, Section 12). Computing the effective scattering amplitude of metric perturbations from the cascade's finite tower and showing it is UV-finite would provide computational confirmation of the structural finiteness.
4. **Decoherence of metric superpositions.** The Born rule applies to metric measurements (Theorem 6.1), but the rate at which macroscopic metric superpositions decohere has not been computed.
5. **Holographic entropy bound.** Boundary dominance ($\Omega/V = d$) is structurally holographic. Whether the Bousso bound follows from the cascade is open.

References

[1] RTAC, *The Cascade Series — Part I: Scale Variance from Orthogonality*, March 2026.

[2] RTAC, *The Cascade Series — Part II: Quantum Mechanics from the Cascade*, March 2026.

[3] RTAC, *The Cascade Series — Part III: General Relativity from the Cascade*, March 2026.

[4] M. H. Goroff and A. Sagnotti, "The ultraviolet behaviour of Einstein gravity," *Nuclear Physics B* **266**, 709–736 (1986).

[5] A. M. Gleason, "Measures on the closed subspaces of a Hilbert space," *Journal of Mathematics and Mechanics* **6**, 885–893 (1957).

[6] D. Lovelock, "The four-dimensionality of space and the Einstein tensor," *Journal of Mathematical Physics* **13**, 874–876 (1972).

[7] B. S. DeWitt, "Quantum theory of gravity. I. The canonical theory," *Physical Review* **160**, 1113–1148 (1967).

[8] S. W. Hawking, "Particle creation by black holes," *Communications in Mathematical Physics* **43**, 199–220 (1975).