

The Cascade Series — Part I

The Cosmological Constant from the Observer's Frame

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Abstract

Part 0 [1] derives the cascade invariant $I_0 = \Omega_{19} \times \Omega_{217} = 1.2051 \times 10^{-120}$ from pure mathematics. This paper adds one input — the hypothesis that the cascade's geometry is indistinguishable from our universe — and applies two cascade-native sphere-area corrections to convert the pure-number invariant into the dimensionless cosmological constant measured in reduced-Planck units.

The observer at $d = 4$ lives on the S^3 shell of a host 5D black hole, experiencing “time” as an asymptotic fall toward the hole's pseudo-horizon: the cascade's own descent direction (Part II Section 7.1). Time is *locked* as this infall, not a free fourth dimension. Consequently only three dimensions — the angular dimensions of the spatial S^3 — are free to receive cascade content, and the relevant 3-volume at the observer's frame is $\Omega_3 = 2\pi^2$.

Two corrections connect the cascade invariant to the physical ratio: (i) the host-frame rescaling $(\Omega_5/\Omega_7)^2 = 9/\pi^2$ from the cascade's natural reference at $d_0 = 7$ to the observer's host at $d_V = 5$ (both cascade-internal sphere ratios); (ii) the 3D projection factor $\Omega_2/\Omega_3 = 2/\pi$ at the observer's frame, which is the latitudinal integral $\int_0^\pi \sin^2 \theta d\theta = \pi/2$ expressed as a cascade sphere-area ratio and arises because the Morse foliation of S^3 by 2-spheres is the projection from per- S^2 content density (the cascade-native unit at each latitude of the locked-time S^3 slice) to integrated S^3 3-volume. Combined:

$$\frac{\rho_\Lambda}{M_{\text{Pl,red}}^4} = \frac{\Omega_2}{\Omega_3} \cdot \left(\frac{\Omega_5}{\Omega_7}\right)^2 \cdot \Omega_{19}\Omega_{217} = \frac{2}{\pi} \cdot \frac{9}{\pi^2} \frac{\Omega_{19} \Omega_{217}}{\pi^2} = \frac{18}{\pi^3} \frac{\Omega_{19} \Omega_{217}}{\pi^3} = 6.996 \times 10^{-121}.$$

Observed (Planck 2018): $(7.150 \pm 0.13) \times 10^{-121}$. Leading deviation: -2.2% , placing Λ in the descent-dependent population. After the Part 0 Supplement first-order Gram correction $\exp(\sum(1 - C_{d,d+1}^2)) = \exp(0.02108)$, the residual is -0.07% , well inside the 1.9% Planck 1σ uncertainty.

Every factor is a cascade-native sphere-area ratio: six sphere areas ($\Omega_2, \Omega_3, \Omega_5, \Omega_7, \Omega_{19}, \Omega_{217}$), zero imported objects. The QFT comparison in Section 6 traces the “ 10^{120} cosmological constant problem” to three structural disagreements: (i) the number of free dimensions at the observer's frame (4 for QFT, 3 for the cascade, because time is locked), (ii) continuous vs discrete mode spectrum, and (iii) hand-imposed UV cutoff vs Gamma-function terminus.

1 The Hypothesis

Definition 1.1 (Hypothesis). *The infinite-dimensional unit ball, descended to four dimensions, is indistinguishable from our universe.*

This is the single input beyond Part 0. It is tested by every prediction in the series. It is not an identification, not a model, not an approximation. It is a factual claim: no measurement can distinguish the cascade's geometry from the observed universe.

2 The Observer's Frame

The observer at $d = 4$ lives on the S^3 shell of a host 5D black hole (Cover Sheet, “The Thought Experiment”). The 3 angular dimensions of S^3 are the observer's genuinely free spatial dimensions; time is the infall direction toward the hole's pseudo-horizon, asymptotic and never completing — the cascade's own descent parameter under the identification of Part II [2], Section 7.1.

By Part 0's boundary dominance ($\Omega_{d-1}/V_d = d$), the S^3 shell carries 4/5 of the $d = 5$ content. The dimension $d_V = 5$ is the volume maximum of Part 0's tower: the geometrically richest interior in the cascade. Part 0's threshold pair ($\Omega_{19}, \Omega_{217}$) is generated by the natural zero at $d_0 = 7$. The invariant $I_0 = \Omega_{19} \times \Omega_{217}$ is intrinsically referenced to this area maximum. The observer is not at $d_0 = 7$. The observer is at $d_V = 5$, with only 3 free spatial dimensions.

Two cascade-native sphere-area corrections are required to connect the pure-number cascade invariant to the physical cosmological-constant ratio measured in reduced-Planck units. The first rescales between the cascade's natural reference at $d_0 = 7$ and the observer's host at $d_V = 5$; the second projects the cascade's per- S^2 content density onto the observer's 3D spatial slice under locked time.

2.1 Correction 1: the host frame at $d_V = 5$

The ratio of sphere areas at the two maxima:

$$\frac{\Omega_5}{\Omega_7} = \frac{\pi^3}{\pi^4/3} = \frac{3}{\pi}.$$

Applied once per threshold sphere area (both factors of the bilinear invariant pass through the host rescaling; Part 0, Lemma 8.8), this contributes $(\Omega_5/\Omega_7)^2 = 9/\pi^2$.

2.2 Correction 2: projection onto the observer's 3D spatial slice

Time is not a free fourth dimension at the observer's frame. In the cover sheet's thought experiment, the observer sits on the S^3 shell of a host 5D black hole and experiences “time” as the asymptotic fall toward the hole's pseudo-horizon. Part II [2], Section 7.1 makes this identification explicit at the cascade-mathematical level: time is the cascade's own slicing direction, the parameter along which the descent proceeds. The infall is asymptotic — the observer never crosses — so the total “time” available at any finite moment is bounded in the cascade's frame. Time is *locked* into the cascade's descent parameter, not a free direction the observer can choose.

The structural consequence. Cascade content descending from the higher-dimensional layers has only *three* free dimensions to project onto at the observer's frame, not four. Content arriving from the compactified interior layers $d = 5, \dots, 217$ passes through the locked-time direction (the infall coordinate) and lands on the observer's 3-sphere S^3 , which is their entire receiving volume at any given moment of the infall.

The natural 3-volume at the observer's frame is therefore $\Omega_3 = 2\pi^2$, the area of the unit S^3 — a cascade-native quantity (Part 0, Corollary 3.2: sphere area is the unique independent cascade quantity), with no imported cube-volume normalisation.

The projection factor from the Morse foliation. The 3-sphere admits a canonical Morse foliation by 2-spheres at constant latitude: writing $S^3 = \{(\cos \theta, \sin \theta \vec{n}) : \theta \in [0, \pi], \vec{n} \in S^2\}$, each latitude θ carries an equatorial 2-sphere of area $\sin^2 \theta \cdot \Omega_2$. Integrating latitude:

$$\Omega_3 = \int_0^\pi \sin^2 \theta \cdot \Omega_2 d\theta = \frac{\pi}{2} \cdot \Omega_2,$$

which is the elementary identity $\int_0^\pi \sin^2 \theta d\theta = \pi/2$ expressed in sphere-area units. Inverted:

$$\frac{\Omega_2}{\Omega_3} = \frac{4\pi}{2\pi^2} = \frac{2}{\pi}.$$

This is the projection factor. Cascade content arriving at the observer's frame is natively per- S^2 (at each latitude of the S^3 foliation); converting to integrated S^3 3-volume introduces the $\pi/2$ latitudinal integral, and its inverse $2/\pi$ is the factor by which per- S^2 content densities are compressed when spread over the full 3-sphere receiving volume. The conversion is purely in cascade-native sphere-area terms — no cube, no QFT mode-counting measure. Both Ω_2 and Ω_3 are unit-sphere areas at dimensions immediately adjacent to the observer; their ratio is a Gamma-function identity.

Remark 2.1 (Why 3 dimensions, not 4). *The cosmological constant is an energy density, and an energy density is an extensive quantity per receiving volume. The receiving volume at the observer's frame has dimension equal to the number of free dimensions the observer actually has — not the number of spacetime dimensions in the abstract. In the cascade picture, three dimensions are free (the angular dimensions of S^3) and one is locked (time, as the cascade's own descent / the observer's asymptotic infall). The projection is therefore over three dimensions, and the natural 3-volume $\Omega_3 = 2\pi^2$ is the integrated Morse foliation of the per- S^2 content density.*

The appearance that time is free — that the observer can “choose” a 4-volume d^4x reference — is a feature of local flat-space approximations on the observer's patch of S^3 , not of the cascade's global structure. Over any cosmological scale where the cascade's asymptotic infall becomes visible (e.g., in ρ_Λ , which involves the full cascade tower $\Omega_{19}\Omega_{217}$), the locked-time character of the cascade's descent forbids projection onto a free fourth dimension and enforces the 3D projection factor $\Omega_2/\Omega_3 = 2/\pi$.

Section 6 develops the QFT contrast: QFT implicitly assumes time is free (4D Minkowski with unbounded time evolution) and therefore counts modes over 4 free dimensions; the cascade has time locked and therefore projects over only 3. This is the structural origin of the $\pi/2$ mismatch at the observer's frame.

3 The Cosmological Constant

Theorem 3.1.

$$\frac{\rho_\Lambda}{M_{\text{Pl,red}}^4} = \underbrace{\frac{\Omega_2}{\Omega_3}}_{\substack{3D \text{ projection} \\ (\text{locked time})}} \cdot \underbrace{\left(\frac{\Omega_5}{\Omega_7}\right)^2}_{\substack{\text{host frame} \\ d_V \rightarrow d_0}} \cdot \Omega_{19} \Omega_{217} = \frac{2}{\pi} \cdot \frac{9}{\pi^2} \cdot \Omega_{19} \Omega_{217} = \frac{18}{\pi^3} \frac{\Omega_{19} \Omega_{217}}{\pi^3} = 6.996 \times 10^{-121}.$$

Proof. The invariant $I_0 = \Omega_{19} \times \Omega_{217}$ is the product of two threshold sphere areas (Part 0, Theorem 8.5). Each is referenced to $d_0 = 7$. The observer at $d_V = 5$ re-references each through one factor of $\Omega_5/\Omega_7 = 3/\pi$, contributing $(3/\pi)^2 = 9/\pi^2$ (Correction 1, host frame). Cascade content from the compactified higher-dimensional layers then projects onto the observer's 3D spatial slice S^3 , with time locked as the cascade's own descent / the observer's asymptotic infall toward the 5D-BH pseudo-horizon (Correction 2). The projection factor from the per- S^2 content density at each latitude of the S^3 Morse foliation to the integrated S^3 3-volume is $\Omega_2/\Omega_3 = 2/\pi$. Combining both corrections:

$$\frac{\rho_\Lambda}{M_{\text{Pl,red}}^4} = \frac{\Omega_2}{\Omega_3} \cdot \left(\frac{\Omega_5}{\Omega_7}\right)^2 \cdot \Omega_{19} \Omega_{217} = \frac{2}{\pi} \cdot \frac{9}{\pi^2} \times 1.20516 \times 10^{-120} = \frac{18}{\pi^3} \times 1.20516 \times 10^{-120} = 6.996 \times 10^{-121}.$$

□

Observed: $(7.150 \pm 0.13) \times 10^{-121}$ (Planck 2018 $H_0 = 67.36$, $\Omega_\Lambda = 0.6847$; uncertainty $\pm 1.9\%$).

Leading deviation: -2.2% . This places Λ in the *descent-dependent* population of observables (negative sign at leading order), alongside α_s , m_τ/m_μ , v , ℓ_A , and Ω_m^{Bott} , all of which the Part 0 Supplement first-order Gram correction closes from $\sim 2\%$ to sub- 0.5% . Applying the same correction to Λ :

$$\frac{\rho_\Lambda}{M_{\text{Pl,red}}^4} \Big|_{\text{Gram}} = \frac{18}{\pi^3} \Omega_{19} \Omega_{217} \cdot \exp\left(\sum_{d=5}^{216} (1 - C_{d,d+1}^2)\right) = 6.996 \times 10^{-121} \cdot e^{0.02108} = 7.145 \times 10^{-121}.$$

Residual: -0.07% , well inside the $\pm 1.9\%$ Planck 1σ uncertainty. The cosmological constant is not a Tier 1 structural exception; it is a Tier 2 descent-dependent observable that closes to sub-experimental precision through the same first-order correction that closes the other descent-dependent members of the series.

Part 0 matches $\log_{10} |\Omega_{19}\Omega_{217}|$ to within ~ 0.3 without any frame correction. Both frame corrections ($9/\pi^2$ from the host-frame rescaling and $2/\pi$ from the 3D projection under locked time) are Gamma-function identities at specified cascade sphere dimensions, not fits. The host-frame correction is internal to the compactified cascade tower; the 3D projection correction arises where the cascade meets the observer's frame and accounts for the fact that only three of the observer's apparent four spacetime dimensions are genuinely free to receive cascade content.

4 The Formula Reads the Tower

Part 0 establishes exactly four distinguished dimensions (Theorem 7.1). The cosmological constant uses all four in the pure-cascade numerator and adds one further sphere-area ratio — the 3D projection factor from the locked-time S^3 Morse foliation — to convert from the cascade's per- S^2 content density to the observer's integrated 3-sphere 3-volume:

$$\frac{\rho_\Lambda}{M_{\text{Pl,red}}^4} = \underbrace{\frac{\Omega_2}{\Omega_3}}_{\substack{\text{3D projection} \\ \text{(locked time)}}} \cdot \underbrace{\frac{\Omega_5^2 \Omega_{19} \Omega_{217}}{\Omega_7^2}}_{\text{Part 0 bilinear invariant}} = \frac{18 \Omega_{19} \Omega_{217}}{\pi^3}.$$

Every factor is a ratio or product of cascade-native sphere areas. No cubes, no imported QFT measures, no external normalisation conventions.

Sphere	dim	Cascade role	Formula role
Ω_2	2	Observer equatorial 2-sphere	3D projection numerator
Ω_3	3	Observer spatial slice S^3	3D projection denominator
Ω_5	5	Volume maximum region ($d_V = 5$)	Host frame
Ω_7	7	Area maximum region ($d_0 = 7$)	Cascade reference
Ω_{19}	19	Phase boundary ($d_1 = 19$)	Near threshold
Ω_{217}	217	Terminus ($d_2 = 217$)	Far threshold

Frame from the two cascade maxima. Scale from the two thresholds. Projection from the observer's locked-time S^3 slice, expressed as the Morse-foliation ratio Ω_2/Ω_3 . Four distinguished cascade dimensions plus two sphere-areas at the observer's spatial receiving volume — six cascade sphere areas in total, all Gamma-function values.

5 The Discrete Vacuum

Between the two thresholds, the cascade has $217 - 19 = 198$ levels. Each carries a specific sphere area, monotonically decreasing from $\Omega_{18} = 0.59$ to $\Omega_{216} \approx 10^{-120}$. The vacuum is 198 discrete

layers of geometry. The smallest fluctuation is $\Omega_{217} \approx 10^{-120}$: the geometric floor, forced by the Gamma function at $p(d) = \sqrt{\pi}$. The detailed contrast with the continuous divergent-integral picture of quantum field theory is developed in the next section.

6 Comparison with Quantum Field Theory

The cascade and quantum field theory are two different ways of counting vacuum content. They give different answers by roughly 10^{120} — the “cosmological constant problem” in its standard framing. This section argues that the difference decomposes into three distinct structural disagreements, each of which is resolved by a specific feature of the cascade’s architecture: (i) the number of free dimensions at the observer’s frame (4 for QFT, 3 for the cascade, because time is locked as the observer’s asymptotic infall), (ii) the spectrum of modes (continuous divergent integral for QFT, finite discrete tower for the cascade), and (iii) the UV regulator (hand-imposed Planck cutoff for QFT, Gamma-function terminus at $d_2 = 217$ for the cascade).

6.1 How many dimensions does the observer have?

QFT’s zero-point vacuum energy density sums the ground-state energies of every mode of every free field on flat Minkowski space $\mathbb{R}^{3,1}$:

$$\rho_{\Lambda}^{\text{QFT}} = \sum_{\text{fields}} \int \frac{d^3k}{(2\pi)^3} \frac{\hbar\omega_k}{2} = \frac{1}{4\pi^2} \int_0^{k_{\text{max}}} k^2 \sqrt{k^2 + m^2} dk + \dots \quad (1)$$

The integration is over 3-momentum space at fixed time, with the implicit assumption that time is a free fourth dimension: the Hamiltonian H generates unbounded unitary time evolution, and the 4D energy-per-volume ρ_{Λ} is extracted by dividing out an infinite time translation. QFT counts modes over *four* free spacetime dimensions.

The cascade’s analogue is a finite sum:

$$\rho_{\Lambda}^{\text{cascade}} \propto \sum_{d=d_1}^{d_2} \Omega_{d-1} = \sum_{d=19}^{217} \Omega_{d-1}, \quad (2)$$

where Ω_{d-1} is the unit $(d-1)$ -sphere area at cascade layer d (Part 0, Corollary 3.2). And the receiving volume at the observer’s frame is *not* 4-dimensional: time is locked as the cascade’s descent parameter, equivalently the observer’s asymptotic infall toward the host 5D-BH pseudo-horizon (Cover Sheet thought experiment, Part II [2] Section 7.1). Only the three angular dimensions of the observer’s spatial slice S^3 are free. Cascade content from the higher-dimensional layers projects onto those three dimensions — not four.

The numerical cost of this dimensional-accounting difference is exactly the $2/\pi$ factor of Theorem 3.1. In the S^3 Morse foliation by 2-spheres at constant latitude, the elementary integral $\int_0^{\pi} \sin^2 \theta d\theta = \pi/2$ gives

$$\Omega_3 = \frac{\pi}{2} \Omega_2 \quad \iff \quad \frac{\Omega_2}{\Omega_3} = \frac{2}{\pi}.$$

Per- S^2 content density (the natural cascade-frame object at each latitude) and integrated- S^3 3-volume (the observer’s full receiving volume) differ by exactly this $2/\pi$ Morse-integral factor. The factor is cascade-native and sphere-valued throughout; no cube, no imported QFT measure.

The observation is sharp: QFT’s apparent 4-dimensional integration region and the cascade’s 3-dimensional S^3 receiving volume are not two normalisation conventions. They reflect a genuine structural disagreement about whether time is free or locked. If time is free (QFT), vacuum energy accounts over 4D spacetime and no $\pi/2$ projection appears. If time is locked as the cascade’s own descent (cascade), vacuum content accounts only over the spatial S^3 and the $\pi/2$ factor is forced by the Morse foliation.

Remark 6.1 (Time as observer artefact). *The 4-dimensional spacetime picture is the observer’s local approximation: on any patch of S^3 small compared to the cosmological curvature scale and any time interval short compared to the infall timescale, the observer sees flat Minkowski space with a free time direction. QFT is built on this local approximation and is internally consistent within it. The cascade argues that the local approximation is exactly that — local. At the cosmological scales where ρ_Λ lives (involving the full cascade tower $\Omega_{19}\Omega_{217}$), the locked-time character of the cascade’s global descent becomes visible, and the naive 4D accounting overcounts by a factor that resolves exactly to $\pi/2$.*

6.2 Two regulators for the same divergence

The integral (1) diverges at $k \rightarrow \infty$ as k_{\max}^4 . Standard QFT imposes a hand-chosen cutoff, typically $k_{\max} \sim M_{\text{Pl}}$, giving $\rho_\Lambda^{\text{QFT}} \sim M_{\text{Pl}}^4$ and the standard 120-orders-of-magnitude fine-tuning problem. The cutoff is external to the formalism: nothing inside QFT selects $k_{\max} = M_{\text{Pl}}$ as opposed to any other scale; the choice is justified by dimensional analysis and the assumption that quantum gravity will regulate the UV.

The cascade has no divergence to regulate. The sum (2) contains exactly 198 terms, because Part 0 proves (Theorem 6.5, orbit termination; Section 5, the four regimes of $p(d)$) that the cascade has exactly two thresholds and terminates structurally at $d_2 = 217$. Beyond d_2 the decay rate $p(d)$ satisfies $p(d) > \sqrt{\pi}$ and sphere areas collapse below 10^{-120} : the “oblivion regime” where the cascade has no geometric content to contribute. The terminus is a theorem about the Gamma function, not an imposed UV cutoff.

Concretely: QFT’s k_{\max} is a free parameter disguised as a physical input; the cascade’s $d_2 = 217$ is the unique integer at which the recurrence’s second canonical constant $c_2 = \sqrt{\pi}$ is reached by the decay rate (Part 0, Theorem 6.7). The “ 10^{120} fine-tuning” that arises from QFT’s divergent integral does not arise in the cascade because the cascade’s sum is convergent by construction. What appears to be a miraculous near-cancellation in QFT is, in the cascade, the natural value of a finite sum dominated by its smallest term ($\Omega_{217} \approx 10^{-120}$).

6.3 Architectural comparison

	Quantum Field Theory	Cascade
Spacetime topology at the observer	Flat $\mathbb{R}^{3,1}$ (locally assumed)	Compact $S^3 \times$ infall (globally)
Free dimensions at the observer	4 (time free, unbounded H)	3 (time locked as infall)
Mode counting	Continuous $\int \frac{d^3k}{(2\pi)^3}$	Discrete $\sum_{d=19}^{217} \Omega_{d-1}$
Receiving volume	4D spacetime, d^4x	3D spatial S^3 , area $\Omega_3 = 2\pi^2$
Projection factor at the observer	none (time is free)	$\frac{\Omega_2}{\Omega_3} = \frac{2}{\pi}$ (Morse foliation of S^3)
UV regulator	Imposed cutoff $k_{\max} \sim M_{\text{Pl}}$	Structural terminus $d_2 = 217$ (Gamma t)
Number of modes summed	Continuous (divergent)	198 discrete layers
Leading ρ_Λ estimate	$\sim M_{\text{Pl}}^4$	$\sim \Omega_{217} M_{\text{Pl,red}}^4 \sim 10^{-120} M_{\text{Pl,red}}^4$
Fine-tuning required	$\sim 10^{120}$	None

6.4 Reframing the cosmological constant problem

The standard statement of the cosmological constant problem assumes QFT is giving a genuine estimate of ρ_Λ that happens to be 10^{120} too large, so that some unknown physics must

cancel it down to the observed value. The cascade reframes this as three separable structural disagreements, none of which requires a cancellation:

1. *The number of free dimensions is different.* QFT counts modes over 4 free dimensions (flat $\mathbb{R}^{3,1}$, with time unbounded). The cascade counts modes over 3 free dimensions (the angular S^3), because time is locked as the observer’s asymptotic infall. The Morse foliation of S^3 by 2-spheres gives the projection factor $\Omega_2/\Omega_3 = 2/\pi$, i.e., $O(1)$, not $O(10^{120})$. This $\pi/2$ is a geometric cost of the 3D projection; it has nothing to do with fine-tuning.
2. *The spectrum of modes is different.* QFT integrates a continuous mode density $d^3k/(2\pi)^3$ and diverges as k_{\max}^4 . The cascade sums a finite discrete tower of 198 layers and converges by construction. This is the $\sim 10^{120}$ factor. It is not a miracle cancellation in the cascade; it is the structural fact that the cascade’s tower is finite, while QFT’s mode integral is infinite before the cutoff is imposed.
3. *The UV regulator is different.* QFT imposes $k_{\max} \sim M_{\text{Pl}}$ by hand. The cascade terminates at $d_2 = 217$ by theorem — specifically, by Part 0 Theorem 6.7, which identifies d_2 as the unique integer at which the recurrence’s second canonical constant $c_2 = \sqrt{\pi}$ is reached by the decay rate. The terminus has no parameter-counting cost. QFT’s cutoff does.

This does not “solve” the cosmological constant problem in the sense of showing how QFT’s naive answer becomes the cascade’s natural answer through some field-theoretic mechanism. The cascade is not a refinement of QFT’s calculation. It is an alternative architecture for counting vacuum content, grounded in three architectural commitments that QFT does not make: time is locked as the observer’s infall; the spectrum of cascade layers is discrete and finite; the terminus is forced by the Gamma function rather than imposed by hand. The $\pi/2$ projection factor of Theorem 3.1 is the consequence of the first commitment (locked time) and is the only such factor of order unity that the cascade needs at the observer’s frame; the remaining $\sim 10^{120}$ difference between QFT’s naive estimate and the observed value is accounted for by the second and third commitments together.

Remark 6.2 (The $\pi/2$ is Part 0’s own lowest slicing integral). *The Morse-foliation identification $\Omega_3 = (\pi/2)\Omega_2$ is not a plausible conjecture awaiting independent verification. It is the same integral as Part 0’s slicing recurrence at its lowest nontrivial step. Part 0’s recurrence is $V_{d+1}/V_d = \int_{-1}^1 (1-x^2)^{d/2} dx$, and at $d = 1$ this gives*

$$\frac{V_2}{V_1} = \int_{-1}^1 (1-x^2)^{1/2} dx = \frac{\pi}{2}.$$

Under the substitution $x = \cos \theta$, the integrand $(1-x^2)^{1/2}$ becomes $\sin \theta$ and the measure dx becomes $\sin \theta d\theta$, so the integral is $\int_0^\pi \sin^2 \theta d\theta = \pi/2$. This is the Morse foliation of S^3 by 2-spheres: at each latitude θ on S^3 , the equatorial 2-sphere has area $\sin^2 \theta \cdot \Omega_2$, and integrating latitude recovers $\Omega_3 = (\pi/2)\Omega_2$. The two identities — the Part 0 slicing step and the S^3 Morse foliation — are literally the same integral in different coordinates. The $\pi/2$ at the observer’s frame is Part 0’s own lowest recurrence step, applied inversely at the observer’s spatial S^3 slice.

The cascade does not do Kaluza–Klein reduction. *Part II=III makes the structural commitment explicit: gravity is not quantised via semiclassical integration over a background because the metric is never an operator on a fixed background — it is a state property on the cascade geometry. The same commitment governs vacuum energy density. There is no integration of continuous zero-point modes over compactified dimensions, regulated by a hand-imposed cutoff, to be performed and then matched to the cascade’s prediction. The cascade’s slicing descent is the replacement for such semiclassical procedures. The $\pi/2$ projection factor is therefore not the output of a Kaluza–Klein calculation awaiting completion; it is a direct sphere identity in*

Part 0's recurrence, applied at the observer's locked-time S^3 slice under the identification of time with the cascade's own descent direction (Part II [2], Section 7.1).

What remains structurally residual in Part I is unification: the three corrections of Theorem 3.1 (host frame $9/\pi^2$, 3D projection $2/\pi$, Part 0 bilinear invariant $\Omega_{19}\Omega_{217}$) are currently assembled as three separate cascade-native sphere-area identities. Tracing them as a single cascade-internal computation from Part 0's tower all the way down to the observer's frame — using only Part 0's slicing recurrence at each step — has not been written out. This is a verification task inside the cascade's own descent framework, and it is the appropriate residual structural task for Part I. It is not a Kaluza–Klein problem, and it is not a semiclassical reduction; the cascade replaces both, and the verification lives entirely inside the cascade's own architecture.

7 What This Paper Does and Does Not Do

Does:

- Introduces the hypothesis (the single physical input of the series).
- Derives $\rho_\Lambda/M_{\text{Pl,red}}^4 = 18\Omega_{19}\Omega_{217}/\pi^3$ from Part 0's bilinear invariant together with two cascade-native sphere-area corrections: the host frame ($9/\pi^2$) from $d_0 = 7$ to $d_V = 5$, and the 3D projection factor $\Omega_2/\Omega_3 = 2/\pi$ at the observer's S^3 slice under locked time. Leading deviation from Planck 2018: -2.2% ; after the Part 0 Supplement first-order Gram correction, residual -0.07% , inside the $\pm 1.9\%$ Planck 1σ uncertainty.
- Places Λ in the cascade's descent-dependent observable population, closed by the same structural correction as α_s , m_τ/m_μ , v , ℓ_A , and Ω_m^{Bott} .
- Identifies the vacuum as 198 discrete layers with a natural terminus.
- Identifies time as the cascade's locked descent parameter (the observer's asymptotic infall toward the 5D-BH pseudo-horizon), with the consequence that cascade content projects over only 3 free dimensions at the observer's frame.
- Identifies the $\pi/2$ projection factor as the cascade-native sphere-area ratio Ω_2/Ω_3 , which coincides with the latitudinal integral $\int_0^\pi \sin^2 \theta d\theta = \pi/2$ of the Morse foliation of S^3 by 2-spheres.

Does not:

- Derive $d = 4$ from the cascade (used here as empirical fact).
- Derive quantum mechanics, general relativity, the Standard Model, or cosmological parameters beyond Λ .
- Derive the Gram first-order correction from first principles; it is imported from the Part 0 Supplement.
- Give a unified cascade-native derivation of Theorem 3.1. The three factors (host frame $9/\pi^2$, 3D projection $2/\pi$, Part 0 bilinear invariant $\Omega_{19}\Omega_{217}$) are currently assembled as three separate cascade-internal sphere-area identities. Tracing them as a single computation from Part 0's tower down to the observer's S^3 frame — using only Part 0's slicing recurrence — is the outstanding structural task. This is *not* a Kaluza–Klein problem and *not* a semiclassical reduction: as developed in Part II=III, the cascade replaces such procedures rather than performing them. The 3D projection factor $\Omega_2/\Omega_3 = 2/\pi$ itself is not open — it is Part 0's own lowest slicing integral $V_2/V_1 = \int_{-1}^1 (1-x^2)^{1/2} dx = \pi/2$ applied inversely at the observer's locked-time S^3 slice (Remark 6.2). What is open is the unified descent calculation.

Inputs: One axiom (orthogonality, via Part 0). One hypothesis. One empirical fact ($d = 4$).
Zero free parameters.

References

- [1] RTAC, *The Cascade Series — Part 0: Scale Variance from Orthogonality*, 2026.
- [2] RTAC, *The Cascade Series — Part II: Quantum Mechanics from the Cascade*, 2026.