The Cascade Series

RTAC

March 2026

The Thought Experiment

A black hole has a horizon. An infalling observer sees infinite time dilation there. But the black hole evaporates in finite time. These two facts are in tension.

The resolution is asymptotic: there is no sharp horizon. Infalling matter never crosses — from outside, it asymptotically approaches the horizon; from the infaller's perspective, the black hole evaporates and the horizon recedes. Both observers agree: the infaller asymptotically tracks a shrinking shell where the rate of infall balances the rate of evaporation.

This shell is the physical content of the "horizon." On it sits a two-dimensional spatial surface (S^2 for a Schwarzschild hole), and the time direction is the asymptotic balance itself — the direction along which infall and shrinkage compete. What if this shell is a $2 + 1$ dimensional universe?

If a 4D black hole produces a $2 + 1$ shell universe, does a 5D black hole produce a $3+1$ shell universe? Is that us? Are we living on the S^3 horizon of a 5D black hole? If so, time is the forced direction — the compactification that the cascade must undergo, not a coordinate we choose. Moving orthogonally, along the shell, slows progress in this forced direction but can never stop it completely. This is time dilation, from the geometry of a shell.

If yes, then the 5D black hole sits inside a 6D space — another shell of a 7D object. Follow the tower upward: each shell is the horizon of the next. The tower terminates at $d = \infty$: the infinite-dimensional unit ball, which has zero volume, zero surface area, no interior. Geometric nothing. A natural starting point.

That was the end of the intuition. The question was: what happens if you work back down from infinity?

One Infinity, Not Many

The thought experiment begins with an infinity—the infinite time dilation at a black hole horizon—and resolves it asymptotically: the infaller never crosses, because the horizon recedes as the hole evaporates. What looked like a horizon is really a shrinking shell the infaller asymptotically tracks; it is the *infall* that takes forever to complete, not the evaporation. The evaporation is finite. Following the tower upward, the thought experiment then arrives at another infinity: B^∞, the starting point. These are not two infinities. They are the same infinity, viewed from two different positions along its descent.

This is the unifying observation of the series. Every "infinity problem" in physics is a local slice of the same asymptotic boundary. The cosmological constant is $\sim 10^{-120}$

because it is the cascade's natural floor after descending from $d = \infty$. The Standard Model mass hierarchy is the descent rate $\exp(-\Phi)$ measured at different cascade depths. Black hole horizons are where the cascade's descent locally approaches its asymptotic limit—the same limit the rest of the universe is descending toward, just at a steeper local rate. Ultraviolet divergences hit a natural floor at Ω_{217}. The Big Bang is the asymptotic upper edge of the descent we are inside, as unreachable going backward as the horizon is going forward.

Physics has treated these as separate problems for a century and made local fixes to each. The cascade says they are one problem with one resolution: there was only ever one infinity, and it is the infinite-dimensional starting object of the series. Every finite structure we observe—spacetime, matter, forces, couplings, masses—is what that one infinity looks like while it is partway through resolving itself. The universe is not a place where infinity was avoided. It is the unique asymptotic completion of the infinity that had to exist, observed from a particular finite depth.

The Starting Point

The thought experiment arrives at the infinite-dimensional unit ball from above, by following a tower of black hole shells upward. But there is a deeper reason to start there, established in the Prelude: *there is nowhere else to start.*

A theory of everything cannot have inputs. Every input demands an explanation of its origin, and that explanation either requires a deeper theory or is circular. The only starting point that requires no explanation is nothing.

Nothing — taken seriously as a mathematical object — is not featureless. The precondition for any formal system is the distinction between true and false: $0 \neq 1$. Distinction implies orthogonality (zero mutual information between distinguishable states). Orthogonality iterates without bound (no logical obstruction to countable independence). The absence of external scale forces unit norm. The result is the infinite-dimensional unit ball: zero volume, zero surface area, no interior. Structured nothing.

The unit ball is not chosen from a menu of starting points. It is the unique mathematical structure that corresponds to "nothing with the capacity for distinction," derived from the precondition for coherent thought. The series is the derivation of what that structure implies.

The Hypothesis

The infinite-dimensional unit ball, descended to four dimensions, is indistinguishable from our universe.

It is tested by deriving, from the cascade's geometry alone, the cosmological constant, the dimension and signature of spacetime, quantum mechanics, and the Standard Model gauge group and its symmetry breaking, three fermion generations, fermion masses, and gauge coupling constants. It further derives the background cosmological parameters — including a Hubble constant between the two competing measurements and an account of the DESI baryon acoustic oscillation observations without dynamical dark energy. Every prediction is a test of the hypothesis.

The Physical Content

If the hypothesis is correct, then every particle in the Standard Model is a *stable projection of higher-dimensional geometry into 4D spacetime.*

Specifically: the cascade's Bott periodicity anchors fermion generations at fixed dimensions ($d = 5, 13, 21$); Adams' theorem anchors gauge bosons at $d = 12, 13, 14$; the Gamma function's critical points anchor the cascade's structural landmarks ($d_V = 5$, $d_0 = 7$, $d_1 = 19$, $d_2 = 217$). These positions are topologically rigid—they cannot be continuously deformed. What the observer at $d = 4$ perceives as "a particle with mass m" is the projection amplitude of one of these features through the cascade's geometric and topological attenuation channels: $\exp(-\Phi)$ for the decay of sphere areas along the descent, and $(2\sqrt{\pi})^{-n_D}$ for each hairy ball obstruction crossed. Heavier particles project from nearer layers; lighter particles from deeper ones. The mass hierarchy *is* the projection hierarchy.

The projection determines *all* particle properties, not only mass. **Spin** is the Clifford type at the particle's cascade layer: complex spinor ($d \bmod 8 = 5$, from $O(d)$ Bott periodicity) for fermions, Adams vector fields for gauge bosons, the hairy ball zero for the Higgs. **Charge** is determined by which gauge layers the projection passes through: colour from $d = 12$, weak isospin from $d = 13$, hypercharge from $d = 14$. **Chirality** follows from the $\mathbb{Z}_2$ decomposition at even-sphere layers ($\chi(S^{2n}) = 2$). **Mixing angles** arise when the projection crosses multiple generation or gauge layers.

The higher dimensions are not hidden containers of extra content. B^∞ has zero volume, zero area, no interior. What the observer sees as particles, forces, and coupling constants is the *shape of the emptiness*—the structured geometry of nothing, projected onto the observer's shell. Stability is topological: the features cannot be removed because the theorems that position them (Bott, Adams, Poincaré–Hopf, the Gamma function) are rigid.

Particle content: what exists and what does not. The cascade produces exactly the Standard Model—three fermion generations, the gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$, its breaking pattern, and one Higgs doublet. The fourth Bott fermion layer at $d = 29$, suppressed to ~ 0.5 eV by the $d_1 = 19$ phase transition, sits at the neutrino mass scale; whether this provides the neutrino mass mechanism is an open question (Part IVb, Open Question 5). The cascade predicts that no supersymmetric partners exist (the topology has no pairing mechanism), no extra gauge bosons (Adams' theorem is unique), no extra Higgs bosons (one hairy ball zero), no axion ($\theta_{\mathrm{QCD}} = 0$ topologically), and no dark matter particles (the cascade's own geometry provides the missing gravitational content), and no gravitons (the metric is a state property, never promoted to an operator; Part II = III, Section 6). Discovery of any of these would falsify the framework.

The series that follows is the quantitative derivation of this picture.

Predictions

One hypothesis. Zero free parameters. Every prediction below is a test of the hypothesis.

Tier 1 — Exact: Forced by Uniqueness Theorems

Mathematical uniqueness proofs leave no alternative. These are not approximations.

Prediction	Value	Status	Source
Spacetime dimension	d = 4	Confirmed	Lovelock ∩ Clifford (III)
Metric signature	(−,+,+,+)	Confirmed	Propagator + Clifford (III)
Gauge group	SU(3) × SU(2) × U(1)	Confirmed	Adams + Bott (IVa)
Symmetry breaking	SU(2) broken; SU(3), U(1) exact	Confirmed	Hairy ball theorem (IVa)
Fermion generations	Exactly 3	Confirmed	Bott periodicity + d_1=19 (IVa)
Dark energy EoS	w = −1 exactly	Confirmed	Fixed geometric constant (III)
Strong CP phase	$\theta_{\mathrm{QCD}} = 0$	Confirmed	$\pi_3(S^{11}) = \mathbb{Z}_2$ (IVa)
No supersymmetry	—	Confirmed (LHC)	No pairing mechanism (IVa)
No dark matter particles	—	Confirmed (null results)	Geometry provides content (V)
No extra Higgs bosons	—	Confirmed (LHC)	One hairy ball zero (IVa)
No gravitons	—	Not yet testable	Metric is state property, not quantised field (II=III, III)

Tier 2 — Derived: Closed-Form, Zero Free Parameters

Numerical predictions from cascade geometry. Formulas are exact; deviations reflect leading-order truncation.

Observable	Formula	Predicted	Observed	Dev.
ρ_Λ / $M^4_{\mathrm{Pl,red}}$	$18 \cdot \Omega_{19} \cdot \Omega_{217} / \pi^3$	0.6996×10^{-120}	0.7150×10^{-120}	−2.2%
Ω_Λ	$(\pi-1)/\pi$	0.6817	0.685 ± 0.007	−0.5%
Ω_m	$1/\pi$	0.3183	0.315 ± 0.007	+1.1%
Ω_r	$1/(4\pi^7)$	8.28×10^{-5}	8.27×10^{-5}	+0.1%
T_{CMB}	from Ω_r, H_0	2.642 K	2.7255 K	−3.1%
H_0	from ρ_Λ, Ω_Λ	66.78 km/s/Mpc	67.4 ± 0.5	−0.9%
t_0	ΛCDM integral	13.88 Gyr	13.80 ± 0.02	+0.6%
m_H / m_W	$\pi/2$	1.5708	1.559	+0.8%
m_μ / m_e	$\exp(\Delta\Phi) \cdot 2\sqrt{\pi}$	206.50	206.77	+0.13%
m_e	geometric-topological	0.514 MeV	0.511 MeV	+0.6%
m_μ	geometric-topological	106.2 MeV	105.66 MeV	+0.5%
$\alpha_s(M_Z)$ leading	$\alpha(12) \cdot \exp(\Delta\Phi)$	0.1159	0.1179 ± 0.0009	−1.7%
$\sin^2\theta_W$ leading	Radon-Hurwitz ratio	0.2286	0.23121	−1.1%

Tier 3 — Precision: Correction-Family Closures

Seven observables close within experimental error via $\delta\Phi = \alpha(\mathrm{d}^*)/\chi^k$ shifts sourced at Part 0's distinguished dimensions. Three shift-observable pairs reuse the same correction across independent quantities.

Observable	Shift source	Predicted	Observed	Residual
$\alpha_s(M_Z)$	$\alpha(14)/\chi$	0.11792	0.1179 ± 0.0009	$+0.02\sigma$
m_τ / m_μ	$\alpha(14)/\chi$	16.8173	16.8170 ± 0.0011	$+0.24\sigma$
m_τ absolute	$\alpha(19)/\chi$	1776.82 MeV	1776.86 ± 0.12	-0.31σ
$\sin^2\theta_W$	$\alpha(5)/\chi^3$	0.23123	0.23121 ± 0.00004	$+0.40\sigma$
Ω_m	$-\alpha(5)/\chi^3$	0.31474	0.315 ± 0.007	-0.04σ
θ_C (Cabibbo)	$-\alpha(7)/\chi^2$	13.04°	$13.04 \pm 0.05°$	$+0.03\sigma$

Tier 4 — Frontier: Under Active Experimental Test

Specific predictions testable by current or near-future experiments (DESI, Euclid, CMB-S4, SH0ES).

Observable	Predicted	Current data	Status
H_0	66.78 km/s/Mpc (Gram-corrected ≈ 67.5)	Planck: 67.4 · SH0ES: 73.0	Planck-side of tension; incon with SH0ES
r_d (sound horizon)	≈147.75 Mpc	Planck: 147.60 Mpc	Essentially eq Planck; cascad ΛCDM share
DESI DR2 BAO fit	χ^2/n = 2.35 (cascade) vs 1.90 (Planck)	Two shared outliers at z=0.510, z=0.706	Cascade fits s worse than Pl both face sam anomalies
DESI w ≠ −1 signal	w = −1 exactly (structural theorem)	DESI DR2: w ≈ −0.76	Challenges ca and ΛCDM ec no ruler-based explanation

Tier 5 — Provisional: Derivation Incomplete

Results where the argument has acknowledged gaps or needs strengthening.

Observable	Issue
$\Omega_b = 1/(2\pi^2)$	"One unit of content on S^3" argument needs strengthening
n_s, A_s	Primordial spectrum not yet derived
Correction selection rule	Observable-to-source assignment not fully derived from first principles

The Series

Prelude: Why Nothing Has Structure. From $0 \neq 1$ to B^∞ in five steps: distinction → orthogonality → infinite dimensions → unit norm → the infinite-dimensional unit ball. No step introduces a free parameter. No step selects from alternatives. The starting point is established.

Part 0: Scale Variance from Orthogonality. Pure mathematics. The slicing recurrence of the unit ball contains one constant ($\sqrt{\pi}$), whose natural zero generates two thresholds at $d = 19$ and $d = 217$. The Gamma function produces exactly four distinguished dimensions: the volume maximum ($d = 5$), the area maximum ($d = 7$), and the two thresholds. No fifth exists. The cascade invariant $\Omega_{19} \times \Omega_{217} = 1.2051 \times 10^{-120}$ is forced. Every step is a theorem about the Gamma function. No physics enters.

Part I: The Cosmological Constant from the Observer's Frame. The hypothesis enters. We observe four dimensions. The observer at $d = 4$ lives on the S^3 boundary of $d = 5$, the volume maximum. Two observer-frame corrections connect the cascade's pure-number invariant to the physical ratio measured in reduced-Planck units: the host frame correction $(\Omega_5/\Omega_7)^2 = 9/\pi^2$ (from cascade reference $d_0 = 7$ to observer's host $d_V = 5$), and the cube–sphere bridge $\Omega_2/V_3^{\mathrm{cube}} = 4\pi/8 = \pi/2$ at the observer's spatial dimension $d = 3$ (converting cascade sphere-area content to the cube-volume normalisation of the reduced Planck mass):

$$\frac{\rho_\Lambda}{M_{\mathrm{Pl,red}}^4} = \frac{2}{\pi} \cdot \frac{9\,\Omega_{19}\,\Omega_{217}}{\pi^2} = \frac{18\,\Omega_{19}\,\Omega_{217}}{\pi^3} = 6.996 \times 10^{-121}.$$

Observed (Planck 2018): $(7.150 \pm 0.13) \times 10^{-121}$. Leading deviation: -2.2%, placing Λ in the descent-dependent population. After the Part 0 Supplement first-order Gram correction, residual -0.07%, inside the Planck 1σ uncertainty. The vacuum is 198 discrete layers of geometry with a natural floor at $\Omega_{217} \approx 10^{-120}$.

Part II: Quantum Mechanics from the Cascade. Consecutive slicing axes are forced orthogonal, producing complex structure, Hilbert space, the Born rule, and unitary evolution — without quantum postulates.

Part III: General Relativity from the Cascade. The cascade's complex spinor structure intersected with Lovelock uniqueness forces $d = 4$, Lorentzian signature, and Einstein's equation. The dark energy equation of state is $w = -1$ exactly.

Part II = III: Quantum Gravity without Quantising Gravity. The quantum and gravitational projections share the same source, propagator, state space, and hypothesis. Gleason forces the Born rule; Lovelock forces Einstein's equation; both are unique at $d = 4$. The standard QM/GR conflicts dissolve. Black hole entropy $S = A/4$ is hidden geometry from boundary dominance: the factor $1/4$ is $1/d$ at the observer's dimension.

Part IVa: The Standard Model from the Cascade (Gauge Group). Bott periodicity, Adams' theorem, and the hairy ball theorem jointly force $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$, its symmetry breaking pattern, and three fermion generations.

Part IVb: The Standard Model from the Cascade (Masses and Couplings). The geometric-topological factorisation of the fermion mass gives fourteen precision predictions — all sub-3%, zero free parameters.

Part V: Cosmology from the Cascade. The cascade derives $\Omega_m = 1/\pi$, $\Omega_b = 1/(2\pi^2)$, $\Omega_r = 1/(4\pi^7)$, and $H_0 = 66.78$ km/s/Mpc from the Friedmann equation with Part I's observer-corrected $\rho_\Lambda/M_{\mathrm{Pl,red}}^4 = (2/\pi)\,I$. Cascade H_0 sits 0.9% below Planck's central 67.4 at leading order and closes to essentially the Planck value under the Part 0 Supplement Gram first-order correction; it is not compatible with the SH0ES 73.0. The cascade's $r_d \approx 147.75$ Mpc is essentially equal to Planck's 147.60 Mpc, so the cascade and Planck share a ruler; the cascade predicts $w = -1$ as a structural theorem and offers no ruler-based alternative account of the DESI apparent-w signal, which challenges both cascade and ΛCDM in the same way. The Friedmann equation has every coefficient determined by π. Universe age $t_0 = 13.88$ Gyr (+0.6%).

The hypothesis is currently not falsified. Papers II–III and II = III are exact: every result matches observation with no approximation and no discrepancy. Papers I, IV, and V use leading-order approximations; at leading order their predictions separate into two populations — descent-dependent quantities with uniformly negative deviations, geometric quantities with positive deviations. The cosmological constant now lives in the descent-dependent population (-2.2% leading, -0.07% after the Part 0 Supplement Gram first-order correction), alongside α_s, m_τ/m_μ, v, ℓ_A, and Ω_m^{Bott}. Part IVb then derives a structural family of cascade potential shifts $\delta\Phi = \alpha(d^*)/\chi$, sourced at distinguished cascade layers from Part 0's four-dimension tower and weighted by the same Euler characteristic $\chi(S^{2n}) = 2$ that appears in the $2\sqrt{\pi}$ fermion obstruction. Seven Standard Model precision observables close within experimental precision with zero fitted parameters, using shifts sourced at four of Paper 0's distinguished dimensions: $\alpha(14)/\chi$ closes α_s and m_τ/m_μ ($d = 14$, U(1) gauge layer); $\alpha(19)/\chi$ closes m_τ absolute and ℓ_A ($d_1 = 19$, phase transition); $\alpha(5)/\chi^3$ closes $\sin^2\theta_W$ and Ω_m ($d_V = 5$, volume maximum, opposite signs for the two populations); $\alpha(7)/\chi^2$ closes θ_C ($d_0 = 7$, area maximum). All are closed forms in Γ function values; three reuse pairs share the same shift across independent observables. A discrete elastic action on the cascade lattice ($S = \sum(2\alpha(d))^{-1}(\Delta\varphi)^2$ with $\varphi = \ln\Omega_d$) generates this family: distinguished layers act as sources, the Green's function decays as $\alpha(d^*)$, and each independent cascade channel filters the signal by $1/\chi$ (one of two chirality basins selected). Falsification requires finding a precision observable whose correction falls outside $\pm\alpha(d^*)/\chi^k$ at a distinguished layer, or demonstrating that the channel-counting rule (k = number of cascade sectors in the observable) fails for a new quantity.

The thought experiment that opens this page is the physical content of the series' central result. The observer is on the S^3 shell of a 5D black hole, partway through an asymptotic compactification that never completes. The cosmological constant is the cascade's geometry measured from that shell. Time dilation is the local rate of the compactification. Black hole horizons are where the transition locally completes. The cosmological constant and time dilation are proportional, in the ratio $d = 5$. The Gamma function knows how big the universe is because the universe is the unit ball, descended.